INTEGRATION

1 **a** Find $\int (2x + 1)\ \mathrm{d}x$.

b Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 1$ and that $y = 5$ when $x = 1$, find an expression for y in terms of x.

2 Use the given boundary conditions to find an expression for y in each case.

a $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 6x$, $y = 1$ at $x = 2$

b $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - x$, $y = 41$ at $x = 4$

c $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 4x + 1$, $y = 4$ at $x = -3$

d $\frac{\mathrm{d}y}{\mathrm{d}x} = 7 - 5x - x^3$, $y = 0$ at $x = 2$

e $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - \frac{2}{x^2}$, $y = -1$ at $x = \frac{1}{2}$

f $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - \sqrt{x}$, $y = 8$ at $x = 4$

3 The curve $y = \mathrm{f}(x)$ passes through the point (3, 5).

Given that $\mathrm{f}'(x) = 3 + 2x - x^2$, find an expression for $\mathrm{f}(x)$.

4 Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}},$$

and that $y = 7$ when $x = 0$, find the value of y when $x = 4$.

5 The curve $y = \mathrm{f}(x)$ passes through the point $(-1, 4)$. Given that $\mathrm{f}'(x) = 2x^3 - x - 8$,

a find an expression for $\mathrm{f}(x)$,

b find an equation of the tangent to the curve at the point on the curve with x-coordinate 2.

6 The curve $y = \mathrm{f}(x)$ passes through the origin.

Given that $\mathrm{f}'(x) = 3x^2 - 8x - 5$, find the coordinates of the other points where the curve crosses the x-axis.

7 Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x + \frac{2}{x^2},$$

a find an expression for y in terms of x.

Given also that $y = 8$ when $x = 2$,

b find the value of y when $x = \frac{1}{2}$.

8 The curve C with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + kx,$$

where k is a constant.

Given that C passes through the points (1, 6) and (2, 1),

a find the value of k,

b find an equation of the curve.