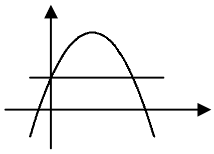
INTEGRATION — *Answers*

1 **a** $f(x) = -[x^2 - 4x] + 3$
$= -[(x-2)^2 - 4] + 3$
$= -(x-2)^2 + 7$
$\therefore\ a = -1,\ b = -2,\ c = 7$

b (2, 7)

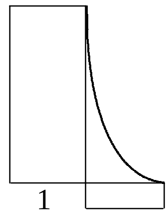
c intersect when
$3 + 4x - x^2 = 3$
$x(4 - x) = 0$
$x = 0, 4$



area below curve
$= \int_0^4 (3 + 4x - x^2)\ dx$
$= [3x + 2x^2 - \frac{1}{3}x^3]_0^4$
$= (12 + 32 - \frac{64}{3}) - 0 = \frac{68}{3}$
area below line
$= 4 \times 3 = 12$
area between line and curve
$= \frac{68}{3} - 12 = 10\frac{2}{3}$

2 **a** $= [-4x^{-2}]_1^2$
$= -1 - (-4)$
$= 3$

b $y = 1 \Rightarrow x = 2$
$y = 8 \Rightarrow x = 1$



shaded area
$= 3 - (1 \times 1) + (7 \times 1)$
$= 9$

3 **a** $\frac{dy}{dx} = 5 - 4x$
grad = 1
$\therefore$ grad of normal = -1
$\therefore\ y - 3 = -(x - 1)$
$[\,y = 4 - x]$

b area below curve
$= \int_0^1 (5x - 2x^2)\ dx$
$= [\frac{5}{2}x^2 - \frac{2}{3}x^3]_0^1$
$= (\frac{5}{2} - \frac{2}{3}) - 0 = \frac{11}{6}$
normal meets y-axis at (0, 4)
area below line
$= \frac{1}{2} \times 1 \times (4 + 3) = \frac{7}{2}$
shaded area
$= \frac{7}{2} - \frac{11}{6} = \frac{5}{3}$

4 **a** $\frac{4 - x^2}{x^2} = 0$
$4 - x^2 = 0$
$x^2 = 4$
$x > 0\ \therefore\ x = 2,\ P\,(2, 0)$

b l: $y - 0 = -3(x - 2)$
$y = 6 - 3x$
intersect when $\frac{4 - x^2}{x^2} = 6 - 3x$
$4 - x^2 = 6x^2 - 3x^3$
$3x^3 - 7x^2 + 4 = 0$
$x = 2$ is a solution $\therefore\ (x - 2)$ is a factor
$(x - 2)(3x^2 - x - 2) = 0$
$(x - 2)(3x + 2)(x - 1) = 0$
$x = 2$ (at P), $-\frac{2}{3}$, 1
$x > 0\ \therefore\ Q\,(1, 3)$

c area below curve
$= \int_1^2 (4x^{-2} - 1)\ dx$
$= [-4x^{-1} - x]_1^2$
$= (-2 - 2) - (-4 - 1) = 1$
area below line
$= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$
area between line and curve
$= \frac{3}{2} - 1 = \frac{1}{2}$