INTEGRATION *Answers*

1 **a** $= [2x^2 - x]_1^3$
$= (18 - 3) - (2 - 1)$
$= 14$

b $= [x^3 + 2x]_0^1$
$= (1 + 2) - (0)$
$= 3$

c $= [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^3$
$= (\frac{9}{2} - 9) - (0)$
$= -\frac{9}{2}$

d $= \int_2^3 (9x^2 + 6x + 1)\ dx$
$= [3x^3 + 3x^2 + x]_2^3$
$= (81 + 27 + 3) - (24 + 12 + 2)$
$= 73$

e $= [\frac{1}{3}x^3 - 4x^2 - 3x]_1^2$
$= (\frac{8}{3} - 16 - 6) - (\frac{1}{3} - 4 - 3)$
$= -12\frac{2}{3}$

f $= [8x - 2x^2 + x^3]_{-2}^4$
$= (32 - 32 + 64) - (-16 - 8 - 8)$
$= 96$

g $= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$
$= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$
$= 27\frac{3}{4}$

h $= [5x + \frac{1}{3}x^3 - x^4]_{-2}^{-1}$
$= (-5 - \frac{1}{3} - 1) - (-10 - \frac{8}{3} - 16)$
$= 22\frac{1}{3}$

i $= [\frac{1}{5}x^5 + 2x^3 - \frac{1}{2}x^2]_{-1}^2$
$= (\frac{32}{5} + 16 - 2) - (-\frac{1}{5} - 2 - \frac{1}{2})$
$= 23\frac{1}{10}$

2 $\int_1^4 (3x^2 + ax - 5)\ dx = [x^3 + \frac{1}{2}ax^2 - 5x]_1^4$
$= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5) = 48 + \frac{15}{2}a$
$\therefore\ 48 + \frac{15}{2}a = 18$
$a = -4$

3 $\int_{-1}^k (3x^2 - 12x + 9)\ dx = [x^3 - 6x^2 + 9x]_{-1}^k$
$= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9) = k^3 - 6k^2 + 9k + 16$
$\therefore\ k^3 - 6k^2 + 9k + 16 = 16$
$k(k^2 - 6k + 9) = 0$
$k(k - 3)^2 = 0$
$k \neq 0 \quad \therefore\ k = 3$

4 **a** $= \int_1^3 (2 - x^{-2})\ dx$
$= [2x + x^{-1}]_1^3$
$= (6 + \frac{1}{3}) - (2 + 1)$
$= \frac{10}{3}$

b $= \int_{-2}^{-1} (6x + 4x^{-3})\ dx$
$= [3x^2 - 2x^{-2}]_{-2}^{-1}$
$= (3 - 2) - (12 - \frac{1}{2})$
$= -10\frac{1}{2}$

c $= [2x^{\frac{3}{2}} - 4x]_1^4$
$= (16 - 16) - (2 - 4)$
$= 2$

d $= \int_{-1}^2 (2x^3 - \frac{1}{2})\ dx$
$= [\frac{1}{2}x^4 - \frac{1}{2}x]_{-1}^2$
$= (8 - 1) - (\frac{1}{2} + \frac{1}{2})$
$= 6$

e $= [\frac{1}{2}x^2 - \frac{3}{2}x^{\frac{2}{3}}]_1^8$
$= (32 - 6) - (\frac{1}{2} - \frac{3}{2})$
$= 27$

f $= \int_2^3 (\frac{1}{3}x^{-2} - 2x)\ dx$
$= [-\frac{1}{3}x^{-1} - x^2]_2^3$
$= (-\frac{1}{9} - 9) - (-\frac{1}{6} - 4)$
$= -4\frac{17}{18}$

5 $= \int_1^3 (3x^2 - 6x + 7)\ dx$
$= [x^3 - 3x^2 + 7x]_1^3$
$= (27 - 27 + 21) - (1 - 3 + 7) = 16$

6 **a** $= \int_0^2 (x^2 + 2)\ dx$

$= [\frac{1}{3}x^3 + 2x]_0^2$

$= (\frac{8}{3} + 4) - 0 = 6\frac{2}{3}$

b $= \int_{-2}^1 (3x^2 + 8x + 6)\ dx$

$= [x^3 + 4x^2 + 6x]_{-2}^1$

$= (1 + 4 + 6) - (-8 + 16 - 12) = 15$

c $= \int_2^4 (9 + 2x - x^2)\ dx$

$= [9x + x^2 - \frac{1}{3}x^3]_2^4$

$= (36 + 16 - \frac{64}{3}) - (18 + 4 - \frac{8}{3}) = 11\frac{1}{3}$

d $= \int_{-1}^0 (x^3 - 4x + 1)\ dx$

$= [\frac{1}{4}x^4 - 2x^2 + x]_{-1}^0$

$= 0 - (\frac{1}{4} - 2 - 1) = \frac{11}{4}$

e $= \int_1^4 (2x + 3x^{\frac{1}{2}})\ dx$

$= [x^2 + 2x^{\frac{3}{2}}]_1^4$

$= (16 + 16) - (1 + 2) = 29$

f $= \int_{-5}^{-1} (3 + 5x^{-2})\ dx$

$= [3x - 5x^{-1}]_{-5}^{-1}$

$= (-3 + 5) - (-15 + 1) = 16$

7 **a** $y = 0 \ \Rightarrow\ 4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore$ $(-2, 0)$ and $(2, 0)$

b $= \int_{-2}^2 (4 - x^2)\ dx$

$= [4x - \frac{1}{3}x^3]_{-2}^2$

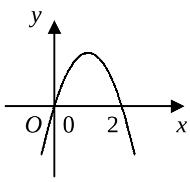
$= (8 - \frac{8}{3}) - (-8 + \frac{8}{3})$

$= 10\frac{2}{3}$

8 **a** $6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0, 2$



area =

$\int_0^2 (6x - 3x^2)\ dx$

$= [3x^2 - x^3]_0^2$

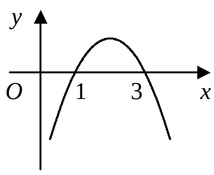
$= (12 - 8) - 0$

$= 4$

b $-x^2 + 4x - 3 = 0$

$-(x - 1)(x - 3) = 0$

$x = 1, 3$



area =

$\int_1^3 (-x^2 + 4x - 3)\ dx$

$= [-\frac{1}{3}x^3 + 2x^2 - 3x]_1^3$

$= (-9 + 18 - 9)$

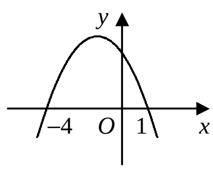
$\quad - (-\frac{1}{3} + 2 - 3)$

$= \frac{4}{3}$

c $4 - 3x - x^2 = 0$

$-(x + 4)(x - 1) = 0$

$x = -4, 1$



area =

$\int_{-4}^1 (4 - 3x - x^2)\ dx$

$= [4x - \frac{3}{2}x^2 - \frac{1}{3}x^3]_{-4}^1$

$= (4 - \frac{3}{2} - \frac{1}{3})$

$\quad - (-16 - 24 + \frac{64}{3})$

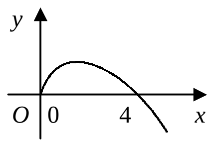
$= 20\frac{5}{6}$

d $2x^{\frac{1}{2}} - x = 0$

$x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$

$x = 0, 4$



area =

$\int_0^4 (2x^{\frac{1}{2}} - x)\ dx$

$= [\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2]_0^4$

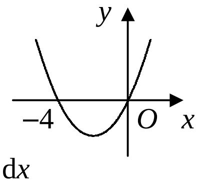
$= (\frac{32}{3} - 8) - 0$

$= \frac{8}{3}$

9 **a** $x^2 + 4x = 0$

$x(x + 4) = 0$

$x = -4, 0$



b $= \int_0^2 (x^2 + 4x)\ dx$

$= [\frac{1}{3}x^3 + 2x^2]_0^2$

$= (\frac{8}{3} + 8) - 0 = 10\frac{2}{3}$

10 **a** $x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5, 3$

$\therefore$ $(-5, 0)$ and $(3, 0)$

b $= [\frac{1}{3}x^3 + x^2 - 15x]_0^3$

$= (9 + 9 - 45) - 0 = -27$

c 27

11 **a** $x^3 - 5x^2 + 6x = 0$

$x(x - 2)(x - 3) = 0$

$x = 0, 2, 3$

$\therefore$ $(0, 0)$, $(2, 0)$ and $(3, 0)$

b $\int_0^2 (x^3 - 5x^2 + 6x)\ dx$

$= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_0^2$

$= (4 - \frac{40}{3} + 12) - 0 = \frac{8}{3}$

$\int_2^3 (x^3 - 5x^2 + 6x)\ dx$

$= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_2^3$

$= (\frac{81}{4} - 45 + 27) - \frac{8}{3} = -\frac{5}{12}$

total area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$

12 **a** $x^2 - 3x + 4 = x + 1$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1, 3$

$\therefore$ $(1, 2)$ and $(3, 4)$

b area below curve

$= \int_1^3 (x^2 - 3x + 4)\ dx$

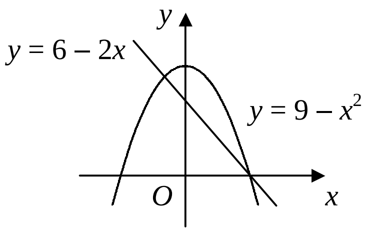
$= [\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x]_1^3$

$= (9 - \frac{27}{2} + 12) - (\frac{1}{3} - \frac{3}{2} + 4) = \frac{14}{3}$

area below line $= \frac{1}{2} \times 2 \times (2 + 4) = 6$

shaded area $= 6 - \frac{14}{3} = \frac{4}{3}$

13 **a**



$9 - x^2 = 6 - 2x$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1, 3$

$\therefore$ intersect at $(-1, 8)$ and $(3, 0)$

area below curve

$= \int_{-1}^3 (9 - x^2)\ dx$

$= [9x - \frac{1}{3}x^3]_{-1}^3$

$= (27 - 9) - (-9 + \frac{1}{3})$

$= 26\frac{2}{3}$

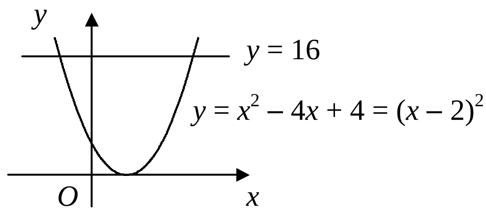
area below line

$= \frac{1}{2} \times 4 \times 8 = 16$

area between line and curve

$= 26\frac{2}{3} - 16 = 10\frac{2}{3}$

b



$x^2 - 4x + 4 = 16$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2, 6$

$\therefore$ intersect at $(-2, 16)$ and $(6, 16)$

area below curve

$= \int_{-2}^6 (x^2 - 4x + 4)\ dx$

$= [\frac{1}{3}x^3 - 2x^2 + 4x]_{-2}^6$

$= (72 - 72 + 24) - (-\frac{8}{3} - 8 - 8)$

$= 42\frac{2}{3}$

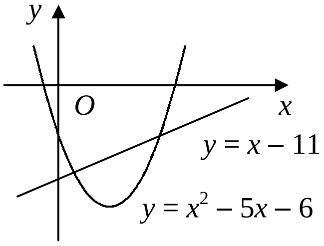
area below line

$= 8 \times 16 = 128$

area between line and curve

$= 128 - 42\frac{2}{3} = 85\frac{1}{3}$

c $y = x^2 - 5x - 6 \Rightarrow y = (x + 1)(x - 6)$



$x^2 - 5x - 6 = x - 11$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$
$x = 1, 5$
$\therefore$ intersect at (1, –10) and (5, –6)
area above curve

$= -\int_1^5 (x^2 - 5x - 6)\ dx$

$= -[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x]_1^5$

$= -[(\frac{125}{3} - \frac{125}{2} - 30) - (\frac{1}{3} - \frac{5}{2} - 6)]$

$= 42\frac{2}{3}$

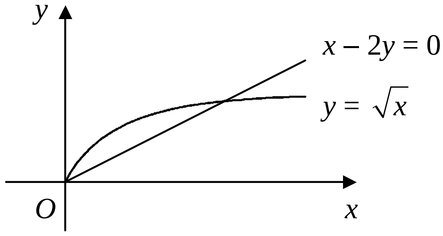
area above line

$= \frac{1}{2} \times 4 \times (10 + 6) = 32$

area between line and curve

$= 42\frac{2}{3} - 32 = 10\frac{2}{3}$

d $x - 2y = 0 \Rightarrow y = \frac{1}{2}x$



$x^{\frac{1}{2}} = \frac{1}{2}x$

$\frac{1}{2}x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$
$x = 0, 4$
$\therefore$ intersect at (0, 0) and (4, 2)
area below curve

$= \int_0^4 x^{\frac{1}{2}}\ dx$

$= [\frac{2}{3}x^{\frac{3}{2}}]_0^4$

$= \frac{16}{3} - 0 = \frac{16}{3}$

area below line

$= \frac{1}{2} \times 4 \times 2 = 4$

area between line and curve

$= \frac{16}{3} - 4 = \frac{4}{3}$