1. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(4)

$$\int_k^9 6x^{-1/2}\,dx = \left[12x^{1/2}\right]_k^9$$

$$20 = \left[12x^{1/2}\right]_k^9 \quad 1$$

$$= \{12(9)^{1/2}\} - \{12(k)^{1/2}\}$$

$$= 36 - 12k^{1/2}$$

$$20 = 36 - 12k^{1/2} \quad 1$$

$$12k^{1/2} = 36 - 20$$

$$= 16$$

$$k^{1/2} = \frac{16}{12} \quad 1$$

$$= \frac{4}{3}$$

$$k = \left(\frac{4}{3}\right)^2$$

$$k = \frac{16}{9} \;\#\quad 1$$

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2. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

$$\int_1^4 (3x^{\frac{1}{2}} + A)\,dx$$

$$= \left[2x^{\frac{3}{2}} + Ax\right]_1^4 \quad - 2$$

$$= \left[2(4)^{\frac{3}{2}} + A(4)\right] - \left[2(1)^{\frac{3}{2}} + A(1)\right]$$

$$= 16 + 4A - 2 - A$$

$$= 14 + 3A$$

$$14 + 3A = 2A^2 \quad - 1$$

$$2A^2 - 3A - 14 = 0$$

$$2A^2 + 4A - 7A - 14 = 0$$

$$2A(A+2) - 7(A+2) = 0$$

$$(A+2)(2A-7) = 0 \quad - 1$$

$$A + 2 = 0 \qquad 2A - 7 = 0$$

$$A = -2 \qquad A = \frac{7}{2} \quad - 1$$

3.

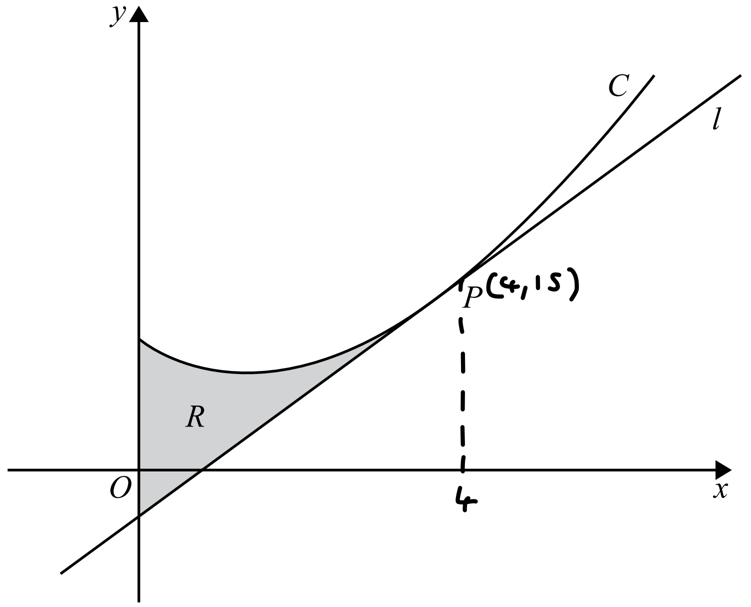

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \; x \geqslant 0$$

The point P with coordinates $(4, 15)$ lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ - 1 - 1

$y - y_1 = m(x - x_1)$

@ $x = 4$

$\frac{15}{2}(4)^{\frac{1}{2}} - 9$ - 1

$= 6$

$y - 15 = 6(x - 4)$

$y - 15 = 6x - 24$

$y = 6x - 9$ - 2

$\int_0^4 (5x^{\frac{3}{2}} - 9x + 11)\,dx - \int_0^4 (6x - 9)\,dx$ - 1

$= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 - \left[3x^2 - 9x\right]_0^4$ - 1

$= \left(\left[2(4)^{\frac{5}{2}} - \frac{9}{2}(4)^2 + 11(4)\right] - 0\right) - \left(\left[3(4)^2 - 9(4)\right] - 0\right)$ - 1

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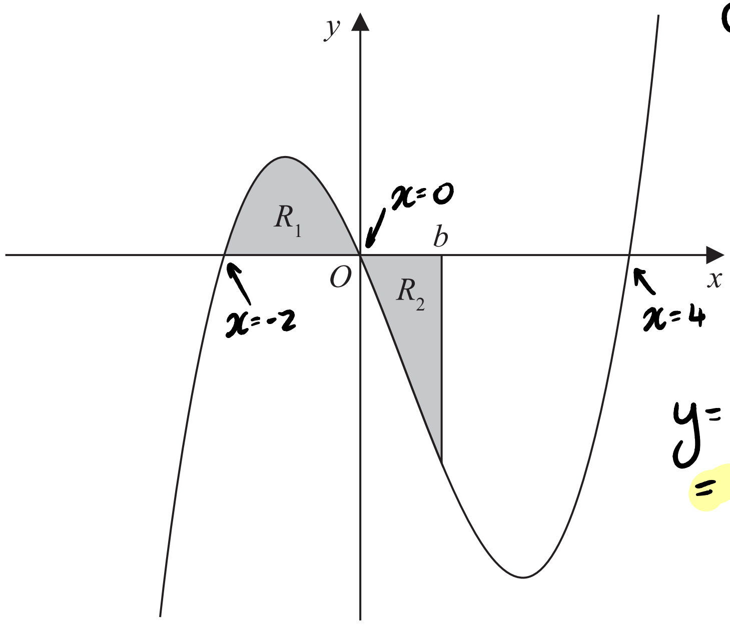
$= 36 - 12$

$= 24$

Area of R = 24 - 1

4.



$0 = x(x+2)(x-4)$

$x=0 \quad x+2=0 \quad x-4=0$

$x=-2 \quad x=4$

$y = x(x^2-2x-8)$

$= x^3-2x^2-8x$

1

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

a) $\int_{-2}^{0} x^3-2x^2-8x \, dx$ 1

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$ 1 *

* $= 0 - \left[\frac{1}{4}(-2)^4 - \frac{2}{3}(-2)^3 - 4(-2)^2\right]$

$= -\left[-\frac{20}{3}\right]$ 1

$= \frac{20}{3}$ as needed

b) $\int_0^b x^3-2x^2-8x\,dx = \frac{-20}{3}$ negative since region is below the x axis

$\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_0^b = \frac{-20}{3}$

$\frac{1}{4}(b)^4-\frac{2}{3}(b)^3-4(b)^2-0 = \frac{-20}{3}$

$(b+2)^2(3b^2-20b+20)=0$

$(b^2+4b+4)(3b^2-20b+20)=0$

$3b^4-20b^3+20b^2+12b^3-80b^2$

$+\cancel{80}b+12b^2-\cancel{80}b+80=0$ (1)

$3b^4-8b^3-48b^2+80=0$

equation B

$\frac{b^4}{4}-\frac{2b^3}{3}-4b^2=\frac{-20}{3}$ (1)

$\times 3$

$\frac{3b^4}{4}-2b^3-12b^2=-20$

$\times 4$

$3b^4-8b^3-48b^2=-80$

$3b^4-8b^3-48b^2+80=0$ (1)

equation A

Since equation A and B are identical we have verified that b satisfies the equation $(b+2)^2(3b^2-20b+20)=0$

(1)

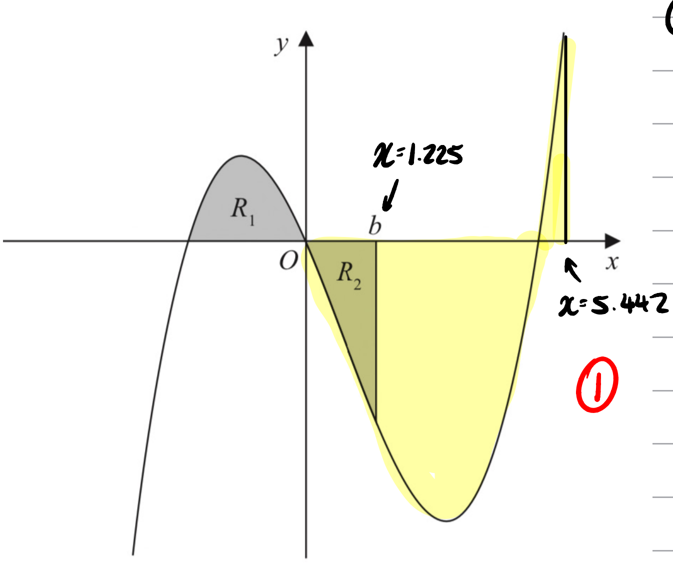


Figure 2

c) $\int_0^b y\,dx = \frac{-20}{3}$

$\int_0^{1.225} y\,dx = \frac{-20}{3}$

$\int_0^{5.442} y\,dx = \frac{-20}{3}$

The net area between 0 and 5.442 is $\frac{-20}{3}$
(The yellow shaded region) ①

5. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

$f(x) = 2x + 3 + 12x^{-2}$

$$\int_1^{2\sqrt{2}} \left(2x + 3 + 12x^{-2}\right) dx = \left[\frac{2x^2}{2} + 3x + \frac{12x^{-1}}{-1}\right]_1^{2\sqrt{2}}$$

$$= \left[x^2 + 3x - \frac{12}{x}\right]_1^{2\sqrt{2}}$$

$$= \left((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12}{2\sqrt{2}}\right) - \left(1^2 + 3(1) - \frac{12}{1}\right)$$

$$= \left(8 + 6\sqrt{2} - \frac{6}{\sqrt{2}}\right) - \left(1 + 3 - 12\right)$$

$$= \left(8 + 6\sqrt{2} - \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}\right) - (-8)$$

$$= \left(8 + 6\sqrt{2} - \frac{6\sqrt{2}}{2}\right) + 8$$

$$= 8 + 6\sqrt{2} - 3\sqrt{2} + 8$$

$$= \boxed{16 + 3\sqrt{2}}$$

(Total for Question 5 is 5 marks)

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6. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) dx$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) dx = 8$$

(3)

a) $\int [4x^{-3} + kx] dx = \frac{4x^{-2}}{-2} + \frac{kx^2}{2} + c$

$= \boxed{\frac{-2}{x^2} + \frac{k}{2}x^2 + c}$

b) $\left[\frac{-2}{x^2} + \frac{k}{2}x^2 \right]_{\frac{1}{2}}^{2} = 8$

$\left[\frac{-2}{4} + \frac{4k}{2} \right] - \left[\frac{-2}{\frac{1}{4}} + \frac{k}{2} \cdot \frac{1}{4} \right] = 8$

$\left[-\frac{1}{2} + \cancel{8} \right] + \left[2k - \frac{k}{8} \right] = \cancel{8}$

$\frac{15}{8}k = \frac{1}{2} \quad \therefore \boxed{k = \frac{4}{15}}$

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