1. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

$f(x) = 2x + 3 + 12x^{-2}$

$$\int_1^{2\sqrt{2}} \left(2x + 3 + 12x^{-2}\right) dx = \left[\frac{2x^2}{2} + 3x + \frac{12x^{-1}}{-1}\right]_1^{2\sqrt{2}}$$

$$= \left[x^2 + 3x - \frac{12}{x}\right]_1^{2\sqrt{2}}$$

$$= \left((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12}{2\sqrt{2}}\right) - \left(1^2 + 3(1) - \frac{12}{1}\right)$$

$$= \left(8 + 6\sqrt{2} - \frac{6}{\sqrt{2}}\right) - \left(1 + 3 - 12\right)$$

$$= \left(8 + 6\sqrt{2} - \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}\right) - (-8)$$

$$= \left(8 + 6\sqrt{2} - \frac{6\sqrt{2}}{2}\right) + 8$$

$$= 8 + 6\sqrt{2} - 3\sqrt{2} + 8$$

$$= \boxed{16 + 3\sqrt{2}}$$

(Total for Question 1 is 5 marks)

Answer ALL questions. Write your answers in the spaces provided.

2. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)dx$$

giving your answer in its simplest form.

(4)

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)dx = \frac{\frac{2}{3}x^4}{4} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + x + C$$

$$= \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + C$$

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3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) dx$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) dx = 8$$

(3)

a) $\int [4x^{-3} + kx] dx = \frac{4x^{-2}}{-2} + \frac{kx^2}{2} + c$

$= \boxed{-\frac{2}{x^2} + \frac{k}{2}x^2 + c}$

b) $\left[-\frac{2}{x^2} + \frac{k}{2}x^2 \right]_{\frac{1}{2}}^{2} = 8$

$\left[-\frac{2}{4} + \frac{4k}{2} \right] - \left[-\frac{2}{\frac{1}{4}} + \frac{k}{2} \cdot \frac{1}{4} \right] = 8$

$\left[-\frac{1}{2} + \cancel{8} \right] + \left[2k - \frac{k}{8} \right] = \cancel{8}$

$\frac{15}{8}k = \frac{1}{2} \quad \therefore \boxed{k = \frac{4}{15}}$

4. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

(4)

a) $\int_1^k \left(\frac{5}{2}x^{-\frac{1}{2}} + 3\right) dx = \left[\left(\frac{5}{2} \div \left(-\frac{1}{2}+1\right)\right)x^{-\frac{1}{2}+1} + \frac{3}{1}x^1\right]_1^k$

$= \left[\left(\frac{5}{2} \div \frac{1}{2}\right)x^{\frac{1}{2}} + 3x\right]_1^k$

$= \left[5x^{\frac{1}{2}} + 3x\right]_1^k$

$= 5\sqrt{k} + 3k - 5 - 3$

$\Rightarrow \quad 4 = 5\sqrt{k} + 3k - 8$

$0 = 5\sqrt{k} + 3k - 12$

b) use $x = \sqrt{k}$: $3x^2 + 5x - 12 = 0$

$(3x-4)(x+3) = 0$

$\Rightarrow x = \sqrt{k} = \frac{4}{3}$ or $\sqrt{k} = -3$

reject negative root so $\underline{k = \frac{16}{9}}$

$\int_1^9 \left(\frac{5}{2}x^{-\frac{1}{2}} + 3\right) dx = \left[5\sqrt{x} + 3x\right]_1^9$

$= 5(3) + 3(9) - 5 - 3$

takes +ve root

$= 34 \neq 4$

5. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

$f'(x) = 6x^2 + ax - 23$

Integration: $\int 6x^2 \, dx = \frac{6x^3}{3} = 2x^3$

$\int ax \, dx = \frac{ax^2}{2}$

$\int -23 \, dx = -23x$

$\Rightarrow f(x) = \int f'(x) \, dx$ 1

$\Rightarrow f(x) = \int 6x^2 + ax - 23 \, dx$

Constant of integration.

$f(x) = 2x^3 + \frac{ax^2}{2} - 23x + C$ 1

Y intercept (Y when x = 0)

$\Rightarrow x = 0,\ f(0) = 2(0)^3 + \frac{a \cdot 0^2}{2} - 23(0) + C = -12$

$\Rightarrow C = -12$ 1

$\Rightarrow f(x) = 2x^3 + \frac{ax^2}{2} - 23x - 12$

$\Rightarrow$ If $x+4$ is a factor of $f(x)$ then $f(-4) = 0$

$\Rightarrow f(-4) = 0 = 2(-4)^3 + \frac{a(-4)^2}{2} - 23(-4) - 12$ 1

$\Rightarrow 0 = -128 + 8a + 80$

$\Rightarrow 8a = 48 \Rightarrow a = \frac{48}{8} = \underline{6}$ 1 $\Rightarrow f(x) = \underline{\underline{2x^3 + 3x^2 - 23x - 12}}$ 1

6. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right) dx = 2A^2$$

show that there are exactly two possible values for A.

(5)

$$\int_1^4 (3x^{\frac{1}{2}} + A)\,dx$$

$$= \left[2x^{\frac{3}{2}} + Ax\right]_1^4 \quad -2$$

$$= \left[2(4)^{\frac{3}{2}} + A(4)\right] - \left[2(1)^{\frac{3}{2}} + A(1)\right]$$

$$= 16 + 4A - 2 - A$$

$$= 14 + 3A$$

$$14 + 3A = 2A^2 \quad -1$$

$$2A^2 - 3A - 14 = 0$$

$$2A^2 + 4A - 7A - 14 = 0$$

$$2A(A+2) - 7(A+2) = 0$$

$$(A+2)(2A-7) = 0 \quad -1$$

$$A + 2 = 0 \qquad 2A - 7 = 0$$

$$A = -2 \qquad A = \frac{7}{2} \quad -1$$

7. Find

$$\int \frac{3x^4 - 4}{2x^3}\,\mathrm{d}x$$

writing your answer in simplest form.

(4)

$$\int \frac{3x^4 - 4}{2x^3}\,dx$$

$$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx \quad 1$$

$$\int \frac{3}{2}x - 2x^{-3}\,dx \quad 1$$

$$1$$

$$= \frac{3}{4}x^2 + x^{-2} + c \quad \#$$

$$1$$

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