Questions

Q1.

Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(Total for question = 4 marks)

Q2.

Given that *A* is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for *A*.

(5)

(Total for question = 5 marks)

Q3.

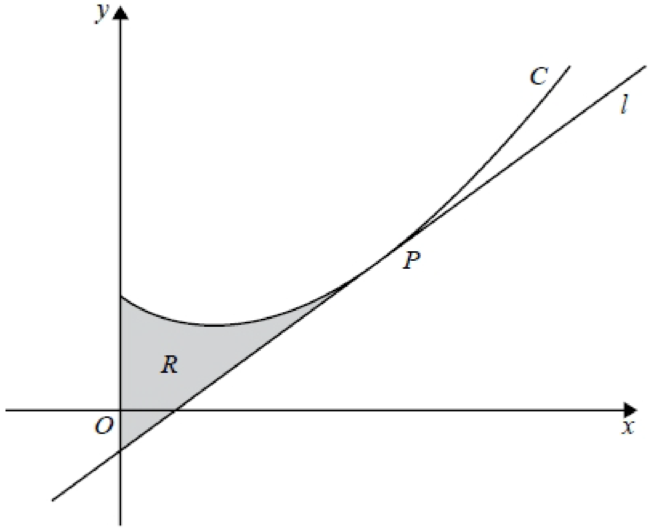


Figure 4

Figure 4 shows a sketch of the curve *C* with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, x \geqslant 0$$

The point *P* with coordinates (4, 15) lies on *C*.

The line *l* is the tangent to *C* at the point *P*.

The region *R*, shown shaded in Figure 4, is bounded by the curve *C*, the line *l* and the *y*-axis.

Show that the area of *R* is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

(Total for question = 10 marks)

Q4.

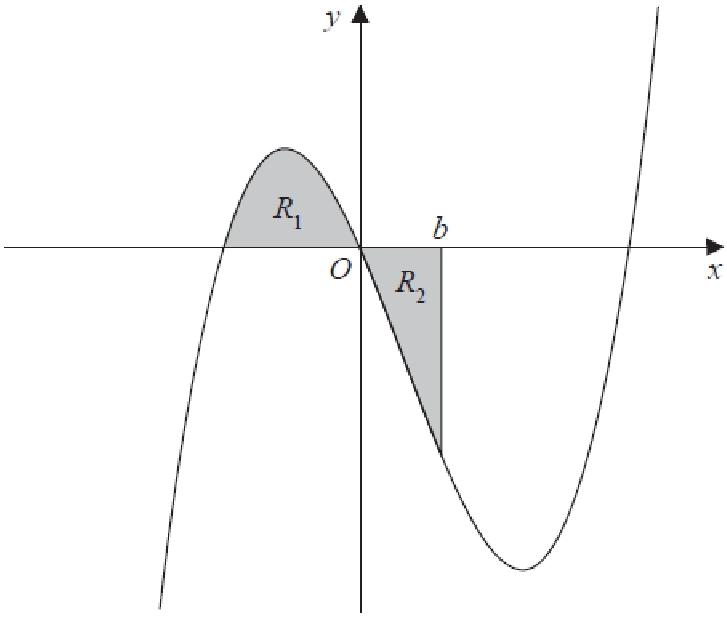


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

(Total for question = 10 marks)

Q5.

Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_{1}^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

(Total for question = 5 marks)

Q6.

(a) Given that *k* is a constant, find

$$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of *k* such that

$$\int_{0.5}^{2}\left(\frac{4}{x^3}+kx\right)\mathrm{d}x=8$$

(3)

(Total for question = 6 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	$\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36 - 12\sqrt{k} = 20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36 - 12\sqrt{k} = 20 \Rightarrow k =$	dM1	3.1a
	$\Rightarrow k = \frac{16}{9}$ oe	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36 - 12\sqrt{k} = 20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}} < 0$

A1: $k = \frac{16}{9}$

Q2.

Question	Scheme		Marks	AOs
	$\int(3x^{0.5}+A)\,\mathrm{d}x = 2x^{1.5}+Ax(+c)$		M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow (2\times8+4A)-(2\times1+A)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
				(5 marks)

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax(+c)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Q3.

Question	Scheme	Marks	AOs
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{15}{2}x^{\frac{1}{2}}-9$	M1 A1	3.1a 1.1b
	Substitutes $x=4\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y-15=6(x-4)$	M1	2.1
	Equation of l is $y=6x-9$	A1	1.1b
	Area $R=\int_0^4\left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,\mathrm{d}x$	M1	3.1a
	$=\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x\right]_0^4=2\times4^{\frac{5}{2}}-\frac{15}{2}\times4^2+20\times4-0$	M1	2.1
	Area of $R=24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
			(10 marks)

Notes:

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

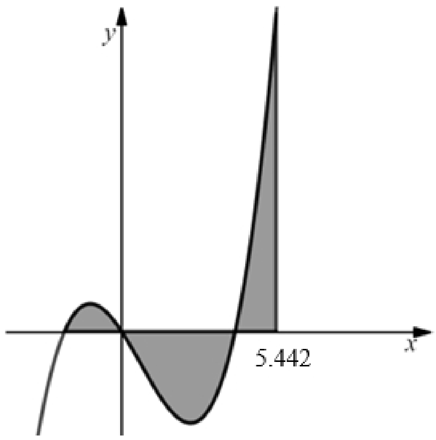
M1: Area under curve $= \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx\right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R = \text{their } 36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

Q4.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	This mark is given for expanding brackets as a first step to a solution
	$\int_{-2}^{0} x^3 - 2x^2 - 8x \ \mathrm{d}x$	M1	This mark is given for a method to find the exact are of R_1
	$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$	M1	This mark is given for a method to evaluate the integral
	$= 0 - (4 - \frac{-16}{3} - 16) = \frac{20}{3}$	A1	This mark is given for a full method to show the exact value of R_1
(b)	$\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = -\frac{20}{3}$	M1	This mark is given for deducing the area of $R_2 = -\frac{20}{3}$
	$3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	This mark is given for rearranging the equation to a quartic
	$(b+2)^2(3b^2 - 20b + 20)$ $= (b^2 + 4b + 4)(3b^2 - 20b + 20)$	M1	This mark is given for expanding the equation given
	$= 3b^4 - 8b^3 - 48b^2 + 80 = 0$ The two equations are the same, so verified	A1	This mark is for showing, and stating, that the equations are the same
(c)		B1	This mark is given for a sketch of the curve with $b = 5.442$ shown
	Between $x = -2$ and $b = 5.442$, the area above the x-axis is the same as the area below the x-axis	B1	This mark is given for a valid explanation of the significance of the root 5.442

Q5.

Question	Scheme	Marks	AOs
	$f(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int \left(+2x + 3 + \frac{12}{x^2} \right) dx = x^2 + 3x - \frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12(\sqrt{2})}{2\times 2} \right) - (-8)$	M1	1.1b
	$= 16 + 3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes

B1: Correct function with numerical powers
M1: Allow for raising power by one. $x^n \to x^{n+1}$
A1: Correct three terms
M1: Substitutes limits and rationalises denominator
A1*: Completely correct, no errors seen.

Q6.

Question	Scheme	Marks	AOs
(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{4}{x^3} + kx \right) dx = -\frac{2}{x^2} + \frac{1}{2}kx^2 + c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2} + \frac{1}{2}kx^2 \right]_{0.5}^{2} = \left(-\frac{2}{2^2} + \frac{1}{2}k \times 4 \right) - \left(-\frac{2}{(0.5)^2} + \frac{1}{2}k \times (0.5)^2 \right) = 8$	M1	1.1b
	$7.5 + \frac{15}{8}k = 8 \Rightarrow k = ...$	dM1	1.1b
	$k = \frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Notes
Mark parts (a) and (b) as one (a) **M1:** For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2. Condone "unprocessed" values here. Eg. x^{-3+1} and x^{1+1} **A1:** Either term correct (un simplified). Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed. **A1:** Correct (and simplified) with $+c$. Ignore spurious notation e.g. answer appearing with an $\int$ sign or with $\mathrm{d}x$ on the end. Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$ (b) **M1:** For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits. **dM1:** For solving a **linear** equation in k. It is dependent upon the previous M only Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k=$ **A1:** $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$ Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267 Please remember to isw after a correct answer