Questions

Q1.

Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

(Total for question = 5 marks)

Q2.

Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

(Total for question = 4 marks)

Q3.

(a) Given that *k* is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of *k* such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

(Total for question = 6 marks)

Q4.

Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) \mathrm{d}x = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) \mathrm{d}x = 4$$

(4)

(Total for question = 8 marks)

Q5.

A curve C has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

(Total for question = 6 marks)

Q6.

Given that *A* is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right) dx = 2A^2$$

show that there are exactly two possible values for *A*.

(5)

(Total for question = 5 marks)

Q7.

Find

$$\int \frac{3x^4 - 4}{2x^3} \, dx$$

writing your answer in simplest form.

(Total for question = 4 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	$\mathrm{f}(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int \left(+2x + 3 + \frac{12}{x^2} \right) \mathrm{d}x = x^2 + 3x - \frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12(\sqrt{2})}{2 \times 2} \right) - (-8)$	M1	1.1b
	$= 16 + 3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes

B1: Correct function with numerical powers
M1: Allow for raising power by one. $x^n \to x^{n+1}$
A1: Correct three terms
M1: Substitutes limits and rationalises denominator
A1*: Completely correct, no errors seen.

Q2.

Question	Scheme	Marks	AOs
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x=\frac{2}{3}\times\frac{x^4}{4}+\ldots+x$	A1	1.1b
	$=\ldots-6\frac{x^{\frac{3}{2}}}{3/2}+\ldots$	A1	1.1b
	$=\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$	A1	1.1b
		(4 marks)	

Notes

M1: Allow for raising power by one. $x^n\to x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two '**non fractional power**' terms (may be un-simplified at this stage)

A1: Correct '**fractional power**' term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line.
Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6}-4x\sqrt{x}+1x^1+c$

Accept $\frac{x^4-24x^{\frac{3}{2}}+6x}{6}+c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$

Q3.

Question	Scheme	Marks	AOs
(a)	$x^n\to x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x=-\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2}=\left(-\frac{2}{2^2}+\frac{1}{2}k\times4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8\Rightarrow k=\ldots$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
		(6 marks)	

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.
Condone "unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).
Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.
Ignore spurious notation e.g. answer appearing with an $\int$ sign or with $\mathrm{d}x$ on the end.
Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k =$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$
Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267
Please remember to isw after a correct answer

Q4.

Question	Scheme	Marks	AOs
(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=... \Rightarrow k=...$ oe	dM1	1.1b
	$k=\frac{16}{9}$, ~~9~~	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with $+c$ and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $...k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for k. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

Q5.

Via firstly integrating

Question	Scheme	Marks	AOs
	$f'(x) = 6x^2 + ax - 23 \Rightarrow f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	$"c" = -12$	B1	2.2a
	$f(-4) = 0 \Rightarrow 2\times(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \ldots\ (6)$	dM1	1.1b
	$(f(x) =)2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(f(x) =)(x+4)(2x^2 - 5x - 3)$ $(f(x) =)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Integrates $f'(x)$ with two correct indices. There is no requirement for the $+ c$

A1: Fully correct integration (may be unsimplified). The $+ c$ must be seen (or implied by the -12)

B1: Deduces that the constant term is -12

dM1: Dependent upon having done some integration. It is for setting up a linear equation in a by using $f(-4) = 0$

May also see long division attempted for this mark. Need to see a complete method leading to a remainder in terms of a which is then set $= 0$.

For reference, the quotient is $2x^2 + \left(\frac{a}{2} - 8\right)x + 9 - 2a$ and the remainder is $8a - 48$

May also use $(x + 4)(px^2 + qx + r) = 2x^3 + \frac{1}{2}ax^2 - 23x - 12$ and compare coefficients to find p, q and r and hence a. Allow this mark if they solve for p, q and r

Note that some candidates use 2f(x) which is acceptable and gives the same result if executed correctly.

dM1: Solves the linear equation in a or uses p, q and r to find a.

It is dependent upon having attempted some integration and used $f(\pm 4) = 0$ or long division/comparing coefficients with $(x + 4)$ as a factor.

A1cso: For $(f(x) =)2x^3 + 3x^2 - 23x - 12$ oe. Note that "f(x) =" does not need to be seen and ignore any "= 0"

Via firstly using factor

Question	Scheme	Marks	AOs
Alt	$f(x) = (x+4)(Ax^2 + Bx + C)$	M1 A1	1.1b 1.1b
	$f(x) = Ax^3 + (4A + B)x^2 + (4B + C)x + 4C \Rightarrow C = -3$	B1	2.2a
	$f'(x) = 3Ax^2 + 2(4A + B)x + (4B + C)$ and $f'(x) = 6x^2 + ax - 23$ $\Rightarrow A = \ldots$	dM1	3.1a
	Full method to get A, B and C	dM1	1.1b
	$f(x) = (x+4)(2x^2 - 5x - 3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Uses the fact that f(x) is a cubic expression with a factor of $(x + 4)$

A1: For $f(x) = (x + 4)(Ax^2 + Bx + C)$

B1: Deduces that $C = -3$

dM1: Attempts to differentiate either by product rule or via multiplication and compares to $f'(x) = 6x^2 + ax - 23$ to find A.
dM1: Full method to get A, B and C
A1cso: $f(x) = (x+4)(2x^2 - 5x - 3)$ or $f(x) = (x+4)(2x+1)(x-3)$

Q6.

Question	Scheme		Marks	AOs
	$\int (3x^{0.5} + A)\,dx = 2x^{1.5} + Ax(+c)$		M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow (2\times 8 + 4A) - (2\times 1 + A) = 2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts $b^2 - 4ac$	M1	1.1b
	$\Rightarrow A = -2, \frac{7}{2}$ and states that there are two roots	States $b^2 - 4ac = 121 > 0$ and hence there are two roots	A1	2.4
			(5 marks)	

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5} + Ax(+c)$ where k is a non- zero constant
A1: Correct answer but may not be simplified
M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,dx = Ax$ and not $\frac{A^2}{2}$
M1: Sets up quadratic equation in A and either attempts to solve or attempts $b^2 - 4ac$
A1: Either $A = -2, \frac{7}{2}$ and states that there are two roots
Or states $b^2 - 4ac = 121 > 0$ and hence there are two roots

Q7.

Question	Scheme	Marks	AOs
	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2}\quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c\quad$ o.e	A1	1.1b
		(4)	
			(4 marks)

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)\,dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.

Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$