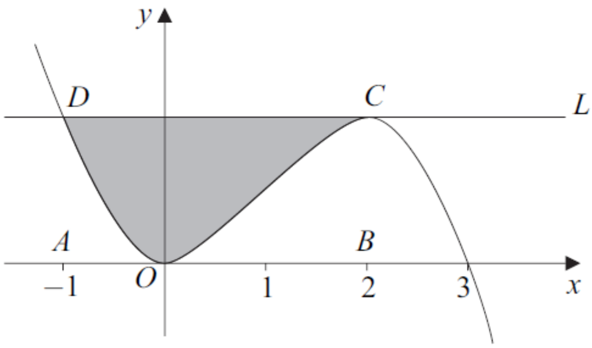
Integration Questions

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{dy}{dx}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

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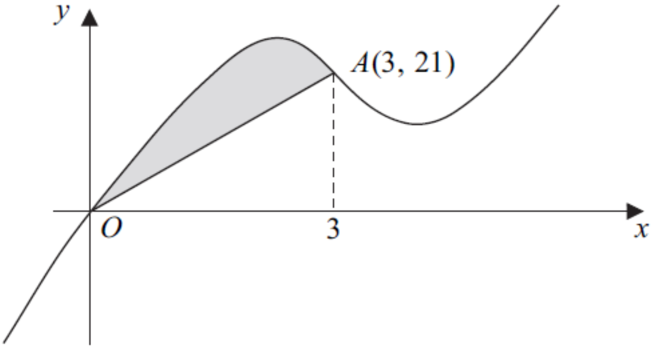
5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3,\ 21)$ lies on the curve.

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\, dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\, dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

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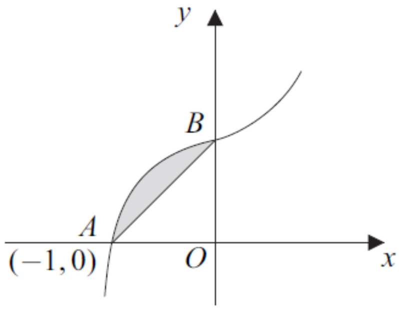
6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

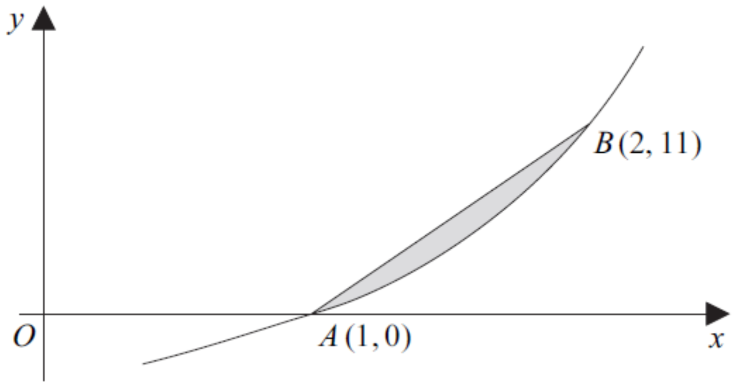
(ii) Find $\int (3x^5 + 2x + 5)\, dx$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\mathrm{d}x$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

Integration Answers

8(a)	$y_D = 3+1 = 4$ or $y_C = 12-8 = 4$	M1		Attempt at either y coordinate
	Area $ABCD = 3\times 4 = 12$	A1	2	
(b)(i)	$x^3 - \frac{x^4}{4} \quad (+C)$	M1 A1 A1	3	Increase one power by 1 One term correct unsimplified All correct unsimplified (condone no +C)
(ii)	Sub limits −1 and 2 into their (b) (i) ans	M1		May use both −1, 0 and 0, 2 instead
	$[8-4]-\left[-1-\frac{1}{4}\right] \quad = 5\frac{1}{4}$	A1		
	Shaded area = "their" (rectangle– integral)	M1		Alt method: difference of two integrals
	$= 12 - 5\frac{1}{4} = 6\frac{3}{4}$	A1	4	**CSO**. Attempted M2, A2
(c)(i)	$\frac{dy}{dx} = 6x - 3x^2$	M1 A1	2	One term correct All correct (no +C etc)
(ii)	When $x = 1$, $y = 2$ when $x = 1$,	B1		May be implied by correct tgt equation
	$\frac{dy}{dx} = 3$ as 'their' grad of tgt	M1√		Ft their derivative when $x = 1$
	Tangent is $y - 2 = 3(x-1)$	A1	3	Any correct form $y = 3x - 1$ etc
(iii)	Decreasing when $\frac{dy}{dx} = 6x - 3x^2 < 0$	M1		Watch no fudging here!! May work backwards in proof.
	$3(2x - x^2) < 0 \quad \Rightarrow x^2 - 2x > 0$	A1	2	**AG** (be convinced no step incorrect)
(d)	Two critical points 0 and 2	M1		Marked on diagram or in solution
	$x > 2$, $x < 0$ ONLY	A1	2	or M1 A0 for $0 < x < 2$ or $0 > x > 2$ **SC** B1 for $x > 2$ (or $x < 0$)
	Total		**18**	

(b)(i)	$\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2 \quad (+c)$	M1 A1 A1	3	One term correct unsimplified Another term correct unsimplified All correct unsimplified (condone missing + c)
(ii)	$\left[\frac{81}{4} - 90 + 126\right] \quad (-0)$	M1		Attempt to sub limit 3 into their (b)(i)
	$= 56\frac{1}{4}$	A1	2	AG Integration, limit sub'n all correct
(iii)	Area of triangle = $31\frac{1}{2}$	B1		Correct unsimplified $\frac{1}{2}\times 21\times 3$
	Shaded Area = $56\frac{1}{4} - \textit{triangle area}$	M1		
	$= 24\frac{3}{4}$	A1	3	Or equivalent such as $\frac{99}{4}$

6(a)(i)	$B\ (0,5)$ Area $AOB = \frac{1}{2}\times 1\times 5$ $= 2\frac{1}{2}$	B1 M1 A1	3	 Condone slip in number or a minus sign
(ii)	$\frac{3x^6}{6}+\frac{2x^2}{2}+5x$ or $\frac{x^6}{2}+x^2+5x$ (may have $+c$ or not)	M1 A1 A1	3	Raise one power by 1 One term correct All correct unsimplified
(iii)	Area under curve $= \int_{-1}^{0} f(x)\,dx$	B1		Correctly written or F(0) – F(–1) correct
	$[0]-\left[\frac{1}{2}+1-5\right]$	M1		Attempt to sub limit(s) of –1 (and 0) Must have integrated
	Area under curve $= 3\frac{1}{2}$	A1		**CSO** (no fudging)
	Area of shaded region $= 3\frac{1}{2}-2\frac{1}{2}=1$	B1√	4	**FT** their integral and triangle (very generous)
(b)(i)	$\frac{dy}{dx}=15x^4+2$	M1 A1		One term correct All correct (no +c etc)
	when $x=-1$, gradient = 17	A1	3	**cso**
(ii)	$y =$ "their gradient"$(x+1)$	B1√	1	Must be finding **tangent** – not normal any form e.g. $y=17x+17$
	Total		**14**	

(b)(i)	$\int \dots\, dx = \frac{x^4}{4}+2x^2-5x\ (+c)$	M1 A1 A1	3	one term correct unsimplified second term correct unsimplified all correct unsimplified
(ii)	$[4+8-10]-\left[\frac{1}{4}+2-5\right]$	M1		correct use of limits 1 and 2; $F(2)-F(1)$ attempted
	$=4\frac{3}{4}$	A1		
	Area of $\Delta = \frac{1}{2}\times 11 = 5\frac{1}{2}$	B1		correct unsimplified
	$\Rightarrow$ shaded area $=5\frac{1}{2}-4\frac{3}{4}$			combined integral of $7x-6-x^3$ scores M1 for limits correctly used then
	$=\frac{3}{4}$	A1	4	A3 correct answer with all working correct