EXPONENTIALS AND LOGARITHMS — *Answers*

1 **a** $= \log_{10} \frac{3}{2}$

$= \log_{10} 3 - \log_{10} 2$

$= b - a$

b $= \log_{10} (2^3 \times 3)$

$= 3 \log_{10} 2 + \log_{10} 3$

$= 3a + b$

c $= \log_{10} (1.5 \times 100)$

$= \log_{10} 1.5 + \log_{10} 100$

$= b - a + 2$

2 **a** $\log_3 x = \frac{5}{4}$

$x = 3^{\frac{5}{4}} = 3.95$ (3sf)

b $3 \log_3 x - 5 \log_3 x = 4$

$\log_3 x = -2$

$x = 3^{-2} = \frac{1}{9}$

3 **a** **i** $= \log_2 q^{\frac{1}{2}} = \frac{1}{2} \log_2 q = \frac{1}{2} p$

ii $= \log_2 8 + \log_2 q = 3 + p$

b $3 + p - \frac{1}{2} p = 2$

$p = \log_2 q = -2$

$\therefore\ q = 2^{-2} = \frac{1}{4}$

4 $2000 = 1000 \times 1.022^{4t}$

$2 = 1.022^{4t}$

$4t \lg 1.022 = \lg 2$

$t = \frac{\lg 2}{4 \lg 1.022} = 7.96$

$\therefore$ 8 years

5 **a** $(0, -3)$

b $k = -4$

c $(\frac{1}{3})^x - 4 = 0$

$(\frac{1}{3})^x = 4$

$x = \frac{\lg 4}{\lg \frac{1}{3}} = -1.26$ (3sf)

6 **a** $\log_3 \frac{x+1}{x-2} = 1$

$\frac{x+1}{x-2} = 3$

$x + 1 = 3x - 6$

$x = \frac{7}{2}$

b $(2x + 1) \lg 3 = (x - 4) \lg 2$

$x (\lg 2 - 2 \lg 3) = \lg 3 + 4 \lg 2$

$x = \frac{\lg 3 + 4 \lg 2}{\lg 2 - 2 \lg 3}$

7 **a** **i** $= 2^{-1}(2^x) = \frac{1}{2} t$

ii $= 2(2^{2x}) = 2(2^x)^2 = 2t^2$

b $2t^2 - 7t + 6 = 0$

$(2t - 3)(t - 2) = 0$

$t = 2^x = \frac{3}{2}, 2$

$x = \frac{\lg \frac{3}{2}}{\lg 2}, 1 = 0.585$ (3sf), 1

8 **a** $\log_2 (3x + 5) + 3 = 7$

$3x + 5 = 2^4 = 16$

$x = \frac{11}{3}$

b $\log_2 (x + 1) + \log_2 (3x - 1) = 5$

$(x + 1)(3x - 1) = 2^5 = 32$

$3x^2 + 2x - 33 = 0$

$(3x + 11)(x - 3) = 0$

$x = -\frac{11}{3}, 3$

for real $\log_2 (3x - 1)$, $x > \frac{1}{3}$ $\quad \therefore\ x = 3$

9 **a** $x + 4 = \frac{5}{4}x$

$x = 16$

b $y + 2 = \frac{12}{y+1}$

$(y + 2)(y + 1) = 12$

$y^2 + 3y - 10 = 0$

$(y + 5)(y - 2) = 0$

$y > 0 \;\therefore\; y = 2$

c $\log_y x = \log_2 16 = 4$

10 **a** $t = 0 \;\Rightarrow\; n = 2000$

b $3600 = \frac{18000}{1+8c^{-3}}$

$1 + 8c^{-3} = 5$

$c^{-3} = \frac{1}{2}$

$c^3 = 2$

$c = \sqrt[3]{2}$

c $4000 = \frac{18000}{1+8c^{-t}}$

$1 + 8c^{-t} = \frac{9}{2}$

$c^{-t} = \frac{7}{16}$

$-t = \frac{\lg \frac{7}{16}}{\lg \sqrt[3]{2}}$

$t = 3.578$ weeks = 25 days

11 **a** **i** $\log_8 x^2 = 2\log_8 x = 2y$

ii $y = \log_8 x \;\Rightarrow\; x = 8^y = 2^{3y}$

$\therefore\; \log_2 x = 3y$

b $3(2y) + 3y = 6$

$y = \log_8 x = \frac{2}{3}$

$\therefore\; x = 8^{\frac{2}{3}} = 4$

12 $\log_2 y - \log_2 (3 - 2x) = 1 \;\Rightarrow\; \frac{y}{3-2x} = 2$

$\Rightarrow\; y = 6 - 4x$

$\log_4 xy = \frac{1}{2} \;\Rightarrow\; xy = 4^{\frac{1}{2}} = 2$

sub. $x(6 - 4x) = 2$

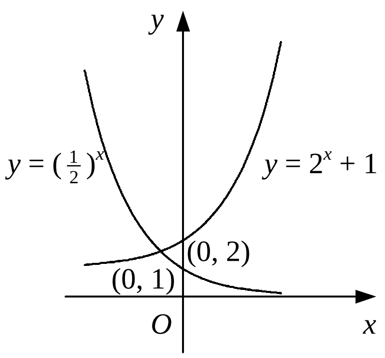
$2x^2 - 3x + 1 = 0$

$(2x - 1)(x - 1) = 0$

$x = \frac{1}{2}, 1$

$\therefore\; x = \frac{1}{2}, y = 4$ or $x = 1, y = 2$

13 **a**



b at A, $2^x + 1 = (\frac{1}{2})^x$

$(2^x)^2 + 2^x = 1$

$2^{2x} + 2^x - 1 = 0$

c $2^x = \frac{-1 \pm \sqrt{1+4}}{2}$

$2^x = \frac{-1-\sqrt{5}}{2}$ [no sols] or $\frac{-1+\sqrt{5}}{2}$

$\therefore\; 2^x = \frac{1}{2}\sqrt{5} - \frac{1}{2}$

$\therefore\; y = (\frac{1}{2}\sqrt{5} - \frac{1}{2}) + 1 = \frac{1}{2}(\sqrt{5} + 1)$

14 **a** when $x = 1$,

LHS = $8 - 4(4) + 2 + 6 = 0$

$\therefore\; x = 1$ is a solution

b $2^{3x} = (2^x)^3 = u^3$

$2^{2x} = (2^x)^2 = u^2$

$\therefore$ (I) $\Rightarrow\; u^3 - 4u^2 + u + 6 = 0$

c $x = 1 \;\Rightarrow\; u = 2 \;\therefore\; (u - 2)$ is a factor

$$
\begin{array}{r|l}
 & u^2 - 2u - 3 \\
u - 2 & u^3 - 4u^2 + u + 6 \\
 & u^3 - 2u^2 \\
 & \quad - 2u^2 + u \\
 & \quad - 2u^2 + 4u \\
 & \qquad - 3u + 6 \\
 & \qquad - 3u + 6
\end{array}
$$

$(u - 2)(u^2 - 2u - 3) = 0$

$(u - 2)(u - 3)(u + 1) = 0$

$u = 2^x = -1$ [no sols], 2 or 3

$x = 1$ (given) or $\frac{\lg 3}{\lg 2} = 1.58$