EXPONENTIALS AND LOGARITHMS — *Answers*

1 **a** $42 = 60e^{100k}$

$100k = \ln 0.7$

$k = \frac{1}{100}\ln 0.7 = -0.00357$ (3sf)

b $30 = 60e^{kt}$

$kt = \ln 0.5$

$t = \frac{100\ln 0.5}{\ln 0.7} = 194$ (3sf)

2 **a** $e^{3x} = 5.7$

$x = \frac{1}{3}\ln 5.7 = 0.58$ (2dp)

b $\ln \frac{x}{x-1} = \frac{1}{2}$

$\frac{x}{x-1} = e^{\frac{1}{2}}$

$x = e^{\frac{1}{2}}(x - 1)$

$x(e^{\frac{1}{2}} - 1) = e^{\frac{1}{2}}$

$x = \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}} - 1} = 2.54$ (2dp)

3 **a** $\ln (4x - 3) = 0$

$4x - 3 = 1$

$x = 1 \quad \therefore A\ (1, 0)$

$1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1} \quad \therefore B\ (e^{-1}, 0)$

b $\ln (4x - 3) = 1 + \ln x$

$\ln (4x - 3) - \ln x = 1$

$\ln \frac{4x-3}{x} = 1$

$\frac{4x-3}{x} = e$

$4x - 3 = ex$

$x(4 - e) = 3$

$x = \frac{3}{4-e}$

4 $2e^{2x} - 7e^x + 3 = 0$

$(2e^x - 1)(e^x - 3) = 0$

$e^x = \frac{1}{2}, 3$

$x = \ln \frac{1}{2}, \ln 3$

5 **a** $t = 0 \Rightarrow N = 800$

b $t = 20 \Rightarrow N = 800e^{0.2}$

$= 977$ (nearest unit)

c $800e^{0.01t} > 2000$

$e^{0.01t} > 2.5$

$0.01t > \ln 2.5$

$t > 91.6 \quad \therefore$ 92 days

6 **a** $1 + e^{2x+1} = 10$

$e^{2x+1} = 9$

$2x + 1 = \ln 9$

$x = \frac{1}{2}(-1 + \ln 9)$

$x = -\frac{1}{2} + \ln 3$

b $1 + e^{2x+1} = 3 - e^x$

$e(e^{2x}) + e^x - 2 = 0$

$e^x = \frac{-1 \pm \sqrt{1+8e}}{2e}$

$x = \ln \frac{-1-\sqrt{1+8e}}{2e}$ (not real)

or $\ln \frac{-1+\sqrt{1+8e}}{2e}$

$\therefore x = -0.366$ (3sf)

7 **a** $4x - 1 = e^2$

$x = \frac{1}{4}(e^2 + 1)$

b $7 = e^{1-3y}$

$1 - 3y = \ln 7$

$y = \frac{1}{3}(1 - \ln 7)$

8 **a** $a = 800$

b $7200 = 800e^{2b}$

$b = \frac{1}{2}\ln 9 = \ln 3$

c $1600 = 800e^{t \ln 3}$

$t = \dfrac{\ln 2}{\ln 3} = 0.631$ hours

$\therefore\ 60 \times 0.631 = 38$ minutes

9 **a** $= \dfrac{(x-1)(x-3)}{(x+2)(x-1)}$

$= \dfrac{x-3}{x+2}$

b $\ln (x^2 - 4x + 3) - \ln (x^2 + x - 2) = 1$

$\ln \dfrac{x^2-4x+3}{x^2+x-2} = \ln \dfrac{x-3}{x+2} = 1$

$\dfrac{x-3}{x+2} = e$

$x - 3 = e(x + 2)$

$x(1 - e) = 2e + 3$

$x = \dfrac{2e+3}{1-e}$

10 $e^y + 5 - 9x = 0 \Rightarrow y = \ln (9x - 5)$

sub. $\ln (9x - 5) - \ln (x + 4) = 2$

$\dfrac{9x-5}{x+4} = e^2$

$9x - 5 = e^2(x + 4)$

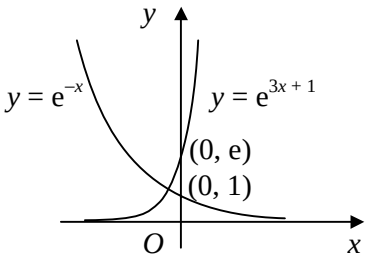
$x(9 - e^2) = 4e^2 + 5$

$x = \dfrac{4e^2+5}{9-e^2} = 21.4509$

$\therefore\ x = 21.5,\ y = 5.24$ (3sf)

11 **a** reflection in y-axis

b



c $e^{-x} = e^{3x+1}$

$1 = e^{4x+1}$

$4x + 1 = 0$

$x = -\frac{1}{4}$

$\therefore\ (-\frac{1}{4},\ e^{\frac{1}{4}})$

12 **a** **i** $= \ln x^{\frac{1}{2}} = \frac{1}{2}\ln x = \frac{1}{2}t$

ii $= \ln e^2 + \ln x = 2 + t$

b $5 + \frac{1}{2}t = 2 + t$

$t = \ln x = 6$

$x = e^6$

13 **a** when $t = 0$, $v = 13$

$\therefore\ 13 = c - 2$

$c = 15$

b $7 = 15e^{-5.1k} - 2$

$e^{-5.1k} = \frac{3}{5}$

$k = \dfrac{\ln \frac{3}{5}}{-5.1} = 0.1002$

c $10 = 15e^{-0.1002t} - 2, \quad 4 = 15e^{-0.1002T} - 2$

$t = \dfrac{\ln \frac{4}{5}}{-0.1002} = 2.2278, \quad T = \dfrac{\ln \frac{2}{5}}{-0.1002} = 9.1481$

$T - t = 6.92$ seconds (3sf)