EXPONENTIALS AND LOGARITHMS

1 A radioactive substance is decaying such that its mass, m grams, at a time t years after initial observation is given by

$$m = 60e^{kt},$$

where k is a constant.

Given that when $t = 100$, $m = 42$,

a find the value of k, **(3)**

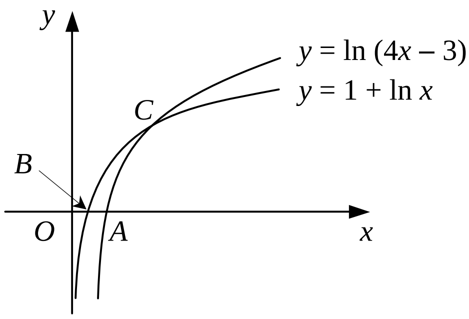
b find the value of t when $m = 30$. **(2)**

2 Solve each equation, giving your answers correct to 2 decimal places.

a $e^{2x} - 5.7e^{-x} = 0$ **(3)**

b $\ln x - \ln (x - 1) = \frac{1}{2}$ **(4)**

3



The diagram shows the curves $y = \ln (4x - 3)$ and $y = 1 + \ln x$ which cross the x-axis at the points A and B respectively.

a Find the coordinates of A and B. **(4)**

The two curves intersect at the point C.

b Find the exact x-coordinate of C, giving your answer in terms of e. **(4)**

4 Find, as natural logarithms, the roots of the equation

$$2e^{x} + 3e^{-x} = 7.$$ **(5)**

5 A scientist carries out an experiment to investigate the growth of a population of flies. She introduces a colony of flies into a closed environment and uses the model that after t days the number of flies in the environment, N, is given by

$$N = 800e^{0.01t}.$$

Find, according to this model,

a the number of flies introduced into the environment, **(1)**

b the size of the population after 20 days, **(2)**

c the least number of days after which the population will exceed 2000. **(3)**

6 $$f(x) = 1 + e^{2x + 1}.$$

a Solve the equation $f(x) = 10$, giving your answer in the form $a + \ln b$ where a is rational and b is an integer. **(3)**

b Find, to 3 significant figures, the x-coordinate of the point where the curve $y = f(x)$ intersects the curve $y = 3 - e^{x}$. **(5)**

7 Giving your answers in exact form, solve the equations

a $\ln (4x - 1) = 2$, **(3)**

b $7 - e^{1 - 3y} = 0$. **(3)**

8 At time $t = 0$, there are 800 bacteria present in a culture. The number of bacteria present at time t hours is modelled by the continuous variable N and the relationship

$$N = ae^{bt},$$

where a and b are constants.

a Write down the value of a. **(1)**

Given that when $t = 2$, $N = 7200$,

b find the value of b in the form $\ln k$, **(3)**

c find, to the nearest minute, how long it takes for the number of bacteria present to double. **(4)**

9 **a** Simplify

$$\frac{x^2 - 4x + 3}{x^2 + x - 2}.$$ **(3)**

b Solve the equation

$$\ln (x^2 - 4x + 3) = 1 + \ln (x^2 + x - 2),$$

giving your answer in terms of e. **(4)**

10 Giving your answers to an appropriate degree of accuracy, solve the simultaneous equations

$$e^y + 5 - 9x = 0$$

$$y - \ln (x + 4) = 2$$ **(7)**

11 **a** Describe fully the single transformation which maps the graph of $y = e^x$ onto the graph of $y = e^{-x}$. **(1)**

b Sketch the graphs of $y = e^{-x}$ and $y = e^{3x + 1}$ on the same diagram, showing the coordinates of any points of intersection with the coordinate axes. **(4)**

c Find the exact coordinates of the point of intersection of the two graphs. **(3)**

12 **a** Given that $t = \ln x$, find expressions in terms of t for

i $\ln \sqrt{x}$,

ii $\ln (e^2x)$. **(4)**

b Hence, or otherwise, solve the equation

$$5 + \ln \sqrt{x} = \ln (e^2x).$$ **(3)**

13 A bead is projected vertically upwards in a jar of liquid with a velocity of 13 m s^{-1}. Its velocity, $v \text{ m s}^{-1}$, at time t seconds after projection, is given by

$$v = ce^{-kt} - 2.$$

a Find the value of c. **(2)**

Given that the bead has a velocity of 7 m s^{-1} after 5.1 seconds, find

b the value of k correct to 4 decimal places, **(3)**

c the time taken for its velocity to decrease from 10 m s^{-1} to 4 m s^{-1}. **(5)**