1. The table shows population data for a country.

Year	1969	1979	1989	1999	2009
Population in millions (p)	58.81	80.35	105.27	134.79	169.71

The data may be represented by an exponential model of growth. Using t as the number of years after 1960, a suitable model is $p = a \times 10^{kt}$.

i. Derive an equation for $\log_{10} p$ 10 in terms of a, k and t.

[2]

ii. Complete the table and draw the graph of $\log_{10} p$ against t, drawing a line of best fit by eye.

[3]

iii. Use your line of best fit to express $\log_{10} p$ in terms of t and hence find p in terms of t.

[4]

iv. According to the model, what was the population in 1960?

[1]

v. According to the model, when will the population reach 200 million?

[3]

2. i. Sketch the graph of $y = 3^x$.

[2]

ii. Solve the equation $3^{5x-1} = 500000$.

[3]

3. A hot drink when first made has a temperature which is 65°C higher than room temperature. The temperature difference, d °C, between the drink and its surroundings decreases by 1.7% each minute.

i. Show that 3 minutes after the drink is made, $d = 61.7$ to 3 significant figures.

[2]

ii. Write down an expression for the value of d at time n minutes after the drink is made, where n is an integer.

[1]

iii. Show that when $d < 3$, n must satisfy the inequality

$$n > \frac{\log_{10}3 - \log_{10}65}{\log_{10}0.983}.$$

Hence find the least integer value of n for which $d < 3$.

[4]

iv. The temperature difference at any time t minutes after the drink is made can also be expressed as $d = 65 \times 10^{-kt}$, for some constant k. Use the value of d for 1 minute after the drink is made to calculate the value of k. Hence find the temperature difference 25.3 minutes after the drink is made.

[4]

4. The thickness of a glacier has been measured every five years from 1960 to 2010. The table shows the reduction in thickness from its measurement in 1960.

Year	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010
Number of years since 1960 (t)	5	10	15	20	25	30	35	40	45	50
Reduction in thickness since 1960 (h m)	0.7	1.0	1.7	2.3	3.6	4.7	6.0	8.2	12	15.9

An exponential model may be used for these data, assuming that the relationship between h and t is of the form $h = a \times 10^{bt}$, where a and b are constants to be determined.

i. Show that this relationship may be expressed in the form $\log_{10}h = mt + c$, stating the values of m and c in terms of a and b.

[2]

ii. Complete the table of values in the answer book, giving your answers correct to 2 decimal places, and plot the graph of $\log_{10}h$ against t, drawing by eye a line of best fit.

[4]

iii. Use your graph to find h in terms of t for this model.

[4]

iv. Calculate by how much the glacier will reduce in thickness between 2010 and 2020, according to the model.

[2]

v. Give one reason why this model will not be suitable in the long term.

[1]

5. Use logarithms to solve the equation $3^{x+1} = 5^{2x}$. Give your answer correct to 3 decimal places.

[4]

6. Fig. 8 shows the graph of $\log_{10}y$ against $\log_{10}x$ It is a straight line passing through the points (2, 8) and (0,2).

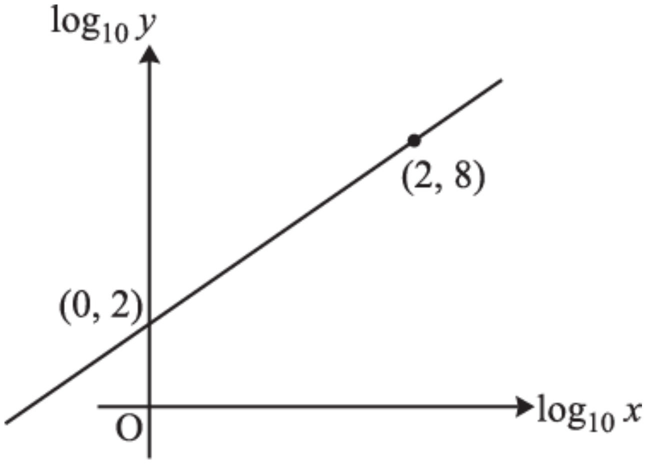


Fig. 8

Find the equation relating $\log_{10}y$ and $\log_{10}x$ and hence find the equation relating y and x.

[4]

7. i. On the same axes, sketch the curves $y = 3^x$ and $y = 3^{2x}$, identifying clearly which is which.

[3]

ii. Given that $3^{2x} = 729$, find in either order the values of 3^x and x.

[2]

8. There are many different flu viruses. The numbers of flu viruses detected in the first few weeks of the 2012–2013 flu epidemic in the UK were as follows.

Week	1	2	3	4	5	6	7	8	9	10
Number of flu viruses	7	10	24	32	40	38	63	96	234	480

These data may be modelled by an equation of the form $y = a \times 10^{bt}$, where y is the number of flu viruses detected in week t of the epidemic, and a and b are constants to be determined.

i. Explain why this model leads to a straight-line graph of $\log_{10} y$ against t. State the gradient and intercept of this graph in terms of a and b.

[3]

ii. Complete the values of $\log_{10} y$ in the table, draw the graph of $\log_{10} y$ against t, and draw by eye a line of best fit for the data.

Hence determine the values of a and b and the equation for y in terms of t for this model.

[8]

During the decline of the epidemic, an appropriate model was

$$y = 921 \times 10^{-0.137w},$$

where y is the number of flu viruses detected in week w of the decline.

iii. Use this to find the number of viruses detected in week 4 of the decline.

[1]

9. i. Simplify $\log_a 1 - \log_a (a^n)^3$.

[2]

ii. Use logarithms to solve the equation $3^{2x+1} = 1000$. Give your answer correct to 3 significant figures.

[3]

10. A biologist is investigating the growth of bacteria in a piece of bread. He believes that the number, N, of bacteria after t hours may be modelled by the relationship $N = A \times 2^{kt}$, where A and k are constants.

(a) Show that, according to the model, the graph of $\log_{10} N$ against t is a straight line. Give, in terms of A and k,
- the gradient of the line
- the intercept on the vertical axis. [4]

The biologist measures the number of bacteria at regular intervals over 22 hours and plots a graph of $\log_{10} N$ against t. He finds that the graph is approximately a straight line with gradient 0.20; the line crosses the vertical axis at 2.0.

(b) Find the values of A and k. [2]

(c) Use the model to predict the number of bacteria after 24 hours. [1]

(d) Give a reason why the model may not be appropriate for large values of t. [1]

11. (a) Express $2\log_3 x + \log_3 a$ as a single logarithm. [1]

(b) Given that $2\log_3 x + \log_3 a = 2$, express x in terms of a. [3]

12. A fisherman has collected statistics for the number of rod-caught salmon in England and Wales. He obtained the following results.

End of year	2004	2005	2006	2007	2008	2009	2015
Number of rod-caught salmon	28 193	21 418	18 776	17 556	16 243	14 526	11 261

Taking y as the number of rod-caught salmon and t as the time in completed years from 2003, the fisherman plotted the graph of $\log_{10} y$ against $\log_{10} t$. This is shown in Fig. 9. The relationship between $\log_{10} y$ and $\log_{10} t$ is modelled by the straight line with equation

$$\log_{10} y = -0.37 \log_{10} t + c.$$

This line is also shown in Fig. 9.

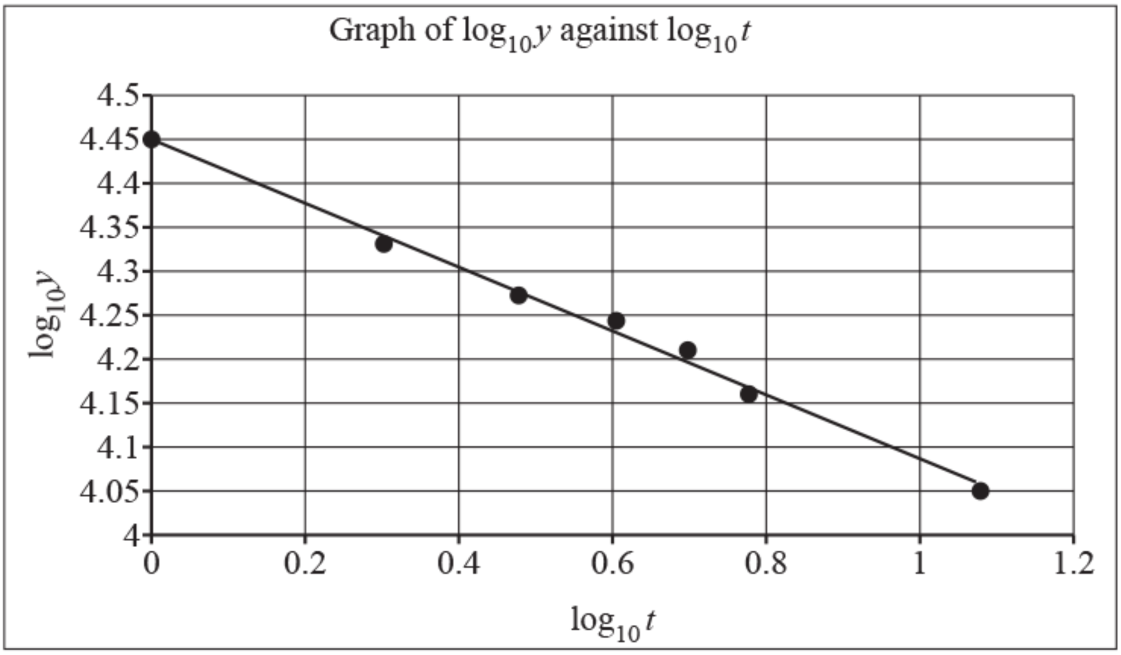


Fig. 9

(a) Use the graph to write down the value of c. [1]

(b) Show that $y \approx 28\,200 t^{-0.37}$. [3]

(c) Verify that the model works well for the year 2006. [1]

(d) Use the model to estimate the number of rod-caught salmon in

(i) 2012, [1]
(ii) 2025. [1]

(e) Comment on the reliability of your answers to part (d). [2]

13. Write down the value of

(a) $\log_a (a^4)$, [1]

(b) $\log_a \left(\frac{1}{a}\right)$ [1]

14. (a) (i) Sketch the graph of $y = 3^x$. [1]

(ii) Give the coordinates of any intercepts. [1]

The curve $y = f(x)$ is the reflection of the curve $y = 3^x$ in the line $y = x$.

(b) Find $f(x)$. [1]

15. In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for t hours. Over the course of the year she realises that her percentage mark for a test, p, may be modelled by the following formula, where A, B and C are constants.

$$p = A - B(t - C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.
- Aaishah had a mark of 22 when she didn't spend any time revising.

(a) Find the values of A, B and C. [3]

(b) According to the model, if Aaishah revises for 45 minutes on the night before the test, what mark will she achieve? [2]

(c) What is the maximum amount of time that Aaishah could have spent revising for the model to work? [2]

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p.

$$p = 22 + 68(1 - e^{-0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
p	59	84	89

Fig. 16

(d) Verify that the data is consistent with the new formula. [2]

(e) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future. Determine whether or not this is good advice. [2]

16. Fig. 7 shows the relationship between the average weekly sales of a newspaper (y) and the time in years after 2010 (x). Each value of y is the average across the whole year. The graph of $\log_{10} y$ plotted against x is approximately a straight line.

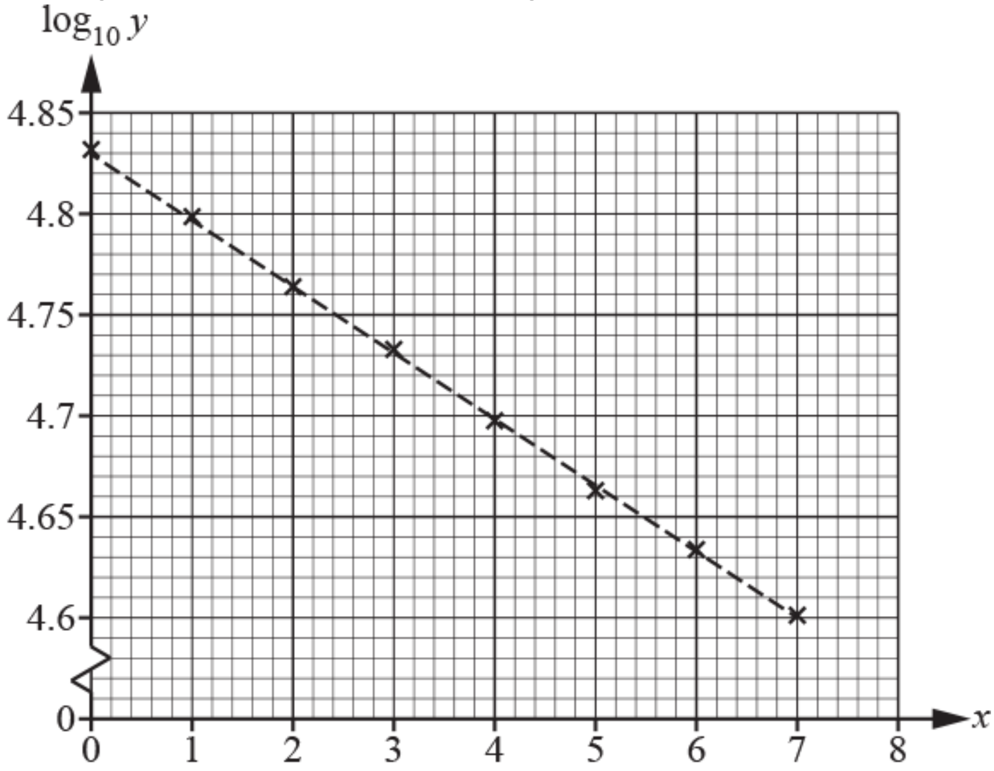


Fig. 7

(a) Show that the straight line is consistent with a model of the form $y = A \times 10^{kx}$, where A and k are constants. [2]

(b) Use the straight line to estimate the values of A and k. Giving the answers correct to 3 significant figures. [4]

(c) Predict the year in which average weekly sales will fall below 10 000. [3]

(d) How reliable do you expect the prediction in part (c) to be? Justify your answer. [1]

17. Write $\log_a x^5 - \log_a\left(\frac{1}{x}\right)$ in the form $k \log_a x$, where k is a constant to be determined. [2]

18. The population of a small country is modelled using the formula $P = 5 \times 1.02^n$ where P is the population in millions and n is the number of years after the start of the year 2000.

(a) According to the model, what is the population of the country at the start of the year 2000? [1]

(b) Explain fully what the model implies about how the population changes over time. [2]

(c) **In this question you must show detailed reasoning.**

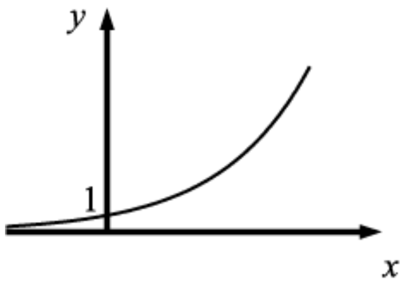
According to the model, in what year will the population reach 10 million? [3]

(d) Show that, according to the model, the graph of $\log_{10}P$ against n will be a straight line. [2]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$\log_{10}p = \log_{10}a + \log_{10}10^{kt}$	M1	condone omission of base;	if unsupported, **B2** for correct equation
		i	$\log_{10}p = \log_{10}a + kt$ www	A1	**Examiner's Comments** The correct equation was often seen, but in many cases it stemmed from wrong working and didn't score. Some candidates stopped at $\log p = \log a + kt\log 10$. $\log p = \log a \times kt$ was a common error; occasionally $\log p = \log a + k\log t$ or logp = loga +logkt were seen.	
		ii	2.02, 2.13, 2.23	B1	allow given to more sig figs	2.022304623 …, 2.129657673, 2.229707433
		ii	plots correct	B1.f.t.	to nearest half square y-intercept between 1.65 and 1.7 and at least one point on or above the line and at least one point on or below the line	
		ii	ruled line of best fit	B1	**Examiner's Comments** This was done very well indeed, with just a few candidates making slips with the plots (usually the middle point), and a few joining each point with a ruler or drawing a curve of best fit to lose the last mark. Only a few candidates lost an easy mark by drawing their line of best fit freehand.	ft their plots must cover range from x = 9 to 49
		iii	0.0105 to 0.0125 for k	B1		must be connected to k
		iii	1.66 to 1.69 for $\log_{10}a$ or 45.7 to 49.0 for a	B1		must be connected to a

		iii	$\log_{10}p$ = their kt + their $\log_{10}a$	B1	must be a correct form for equation of line and with their y-intercept and their gradient (may be found from graph or from table, must be correct method)	
		iii	p = their "$47.9 \times 10^{0.0115t}$" or $10^{1.6785 + 0.0115t}$"	B1	as above, "47.9" and "0.0115" must follow from correct method **Examiner's Comments** Most were able to obtain values for the gradient and the y-intercept within the acceptable range, but not all knew what to do with these. For example, log 1.66 or $10^{1.66}$ were often seen in the equation for log$p$. A surprising number of candidates neglected to include an equation for logp at all, and went straight to an equation for p. This was sometimes correct, even if the equation for logp was incorrect. However, a common error was (for example) p = 45.7 + $10^{0.012t}$.	
		iv	45.7 to 49.0 million	1	'million' needed, not just the value of p **Examiner's Comments** Although many candidates correctly identified the value of loga as crucial in their response, many of them neglected to include the word "million" and lost an easy mark.	
		v	reading from graph at 2.301..	M1*	or $\log_{10}200$ = "$\log_{10}a + kt$"	or 200 = "$10^{\log a + kt}$" oe
		v	their 54	M1dep*	eg for their $t = \dfrac{\log 200 - 1.68}{0.0115}$	or **M1** for their $t = \dfrac{\log \frac{200}{47.9}}{0.0115}$
		v	2014 cao	A1	if unsupported, allow **B3** only if consistent with graph **Examiner's Comments** Most candidates had the sense to revert to their graph. Accurate plotting and a good line of best fit often rewarded them with full	

					marks. However, most candidates used their answer to part (iii) and sometimes lost the final mark due to rounding. A few used 200 000 000 instead of 200 in one of their equations and failed to score.	
			Total	13		
2	i			M1	for curve of correct shape in both quadrants	**SC1** for curve correct in 1st quadrant and touching (0,1) or identified in commentary
	i			A1	through (0, 1) shown on graph or in commentary **Examiner's Comments** This was tackled successfully by most. Most sketches were correct in both quadrants, and (0, 1) was often identified. A small number of candidates only sketched the curve in the first quadrant.	
	ii		$5x - 1 = \dfrac{\log_{10} 500\,000}{\log_{10} 3}$	M1	or $5x - 1 = \log_3 500\ 000$	condone omission of base 10 use of logs in other bases may earn full marks
	ii		$x = (\dfrac{\log_{10} 500\,000}{\log_{10} 3} + 1) \div 5$	M1	$x = (\log_3 500000 + 1) \div 5$ oe; or **B3** www	
	ii		[x =] 2.588 to 2.59	A1	**Examiner's Comments** This was very well done. A correct initial step of log3500 000 or $\log 500\ 000 / \log 3$ was almost always present. The most	if unsupported, **B3** for correct answer to 3 sf or more www

					common error was to then subtract 1 from each side. Occasionally only 1 term was divided by 5, and again some candidates rounded prematurely and lost the final mark.	
			Total	5		
3		i	$65 \times (1 - 0.017)^3$ oe	M1	may be longer method finding decrease year by year etc.	NB use of 3×0.017 leads to 61.685, which doesn't score
		i	61.7410… showing more than 3 sf	A1	answer 61.7 given **Examiner's Comments** A surprising number of candidates failed to score any marks. Many of these candidates adopted a 'simple interest' approach and evaluated $65 - 3\times0.017\times65$. A few candidates evaluated $65 - 3\times0.017$ or wrote $0.017^3\times65 = 61.7$. About two thirds of candidates did understand what was required but failed to appreciate the need to show more than 3 significant figures in their answer to 'show that' the value is 61.7 to this precision. $65\times0.983^3 = 61.7$ was quite common. A significant minority of candidates adopted a long-winded approach, showing each stage of the change, and were no more successful.	
		ii	$[d =]\ 65 \times 0.983^n$ oe	B1	e.g. $63.895 \times 0.983^{n-1}$ or $61.7 \times 0.983^{n-3}$ **Examiner's Comments** Fewer than 40% of candidates earned this mark. $65\times0.983^{n-1}$ was quite common, but more often than not the response was either non-existent or irrelevant.	
		iii	$65 \times 0.983^n < 3$ or $\log_{10}(65 \times 0.983^n) < \log_{10}3$ oe	M1*		condone omission of base 10 throughout
		iii	$\log_{10}65 + \log_{10}0.983^n < \log_{10}3$ www	M1dep	may be implied by e.g. $\log_{10}65 + n\log_{10}0.983 < \log_{10}3$	if **M0M0**, **SC1** for $\log_{10}65 + n\log_{10}0.983 < \log_{10}3$ even if < is replaced by e.g. = or > with no prior incorrect log moves

		iii	$[\log_{10}65 + n\log_{10}0.983 < \log_{10}3]$ $n\log_{10}0.983 < \log_{10}3 - \log_{10}65$ and		or $[\log_{10}0.983^n < \log_{10}3 - \log_{10}65]$	NB watch for correct inequality sign at each step
		iii	completion to $n > \dfrac{\log_{10}3 - \log_{10}65}{\log_{10}0.983}$ **AG** www	A1	inequality signs must be correct throughout	reason for change of inequality sign not required
		iii	n = 180 cao	B1	**B0** for $n > 180$ Examiner's Comments This was inaccessible to most candidates, at least partly due to lack of success in the first two parts. It was surprising how few took advantage of the mark for obtaining n = 180: this request was either ignored, or a decimal answer was presented – although a few wrote $n > 180$. Very few scored all 3 marks for finding the given result. Most who did, had a correct formula from (ii) but had the inequality sign incorrect or used "=". Very few started off correctly, of those who did start correctly, a high proportion lost the third mark for reversing the sign too early. $\log_{10}(65 \times 0.983^n) < \log_{10}3$ very often incorrectly led straight to $\log_{10}(65) \times \log(0.983^n) < \log_{10}3$ which then became $\log_{10}65 + \log_{10}0.983^n < \log_{10}3$. It was pleasing that many of the successful candidates who did score full marks were justifying the reversal of the inequality sign, even though this was not required.	$n > 179.38…$
		iv	$63.895 = 65 \times 10^{-k}$ soi	B1	or $65 \times 0.983 = 65 \times 10^{-k}$	accept 63.895 rot to 3 or 4 sf; **B1** may be awarded for substitution of t = 1 after manipulation
		iv	$\log_{10}$(their 63.895) = $\log_{10}65 - k$ or $-k = \log_{10}$(their 0.983)	M1	their 63.895 must be from attempt to reduce 65 by 1.7% at least once	**M1A1A1** may be awarded if other value of t with correct d is used
		iv	$[k =]\ 7.4 \times 10^{-3}$ to 7.45×10^{-3}	A1	$[k =]\ -\log_{10}0.983$ isw	
		iv	$[d =]$ 42.1… to 42.123 [°C] isw	A1	Examiner's Comments This proved more accessible than part (iii). A little under half of candidates were able to correctly substitute the appropriate value	NB **B1M1A0A1** is possible; unsupported answers for k and / or d do not score

					for d in conjunction with $t = 1$. However, 63.895 = 65 $\times 10^{-k}$ leading to log 63.895 = log65 × log10^{-k} was quite common, so the remaining marks were inaccessible. Some candidates went on to earn the method mark, but lost at least one of the accuracy marks due to premature approximation - some candidates lost a mark by omitting to give an explicit statement of the value of k. Some lost both A marks because they divided by log65 instead of subtracting. A significant minority omitted the question altogether. In cases where there was an attempt which scored zero, the most common error was to begin with $d = 1$.	
			Total	11		
4		i	$\log_{10}h = \log_{10}a + bt$ www	B1		condone omission of base
		i	$m = b$, $c = \log_{10}a$	B1	**Examiner's Comments** Wrong working often spoiled a correct final answer in this question. It was disappointing to see a significant proportion of candidates failing to score both marks on a very standard piece of work.	must be clearly **stated:** linking equations is insufficient
		ii	–0.15, 0[.00], 0.23, 0.36, 0.56, 0.67, 0.78, 0.91, 1.08, 1.2[0]	B2	B1 if 1 error	
		ii	plots correct (tolerance half square)	B1	condone 1 error – see overlay	no ft available for plots
		ii	single ruled line of best fit for values of x from 5 to 50 inclusive	B1	line must not go outside overlay between $x = 5$ and $x = 50$ **Examiner's Comments** This was very well done. A few candidates made errors in the table — usually the first or the penultimate value. A tiny minority gave all values to a different degree of accuracy to the one requested, thus losing two easy marks — although credit was still available for the plots and the line. Most plotted the points adequately and drew a single ruled line of best fit across the whole range of x-values to earn two marks.	
		iii	$-0.3 \le y$-intercept ≤ -0.22	B1	may be implied by $0.5 \le a \le 0.603$	

		iii	valid method to find gradient of line	M1	may be embedded in equation; may be implied by eg m between 0.025 and 0.035 **Examiner's Comments** Those candidates who used their graph to find the gradient and the intercept often went on to score full marks in this part. Those who adopted other methods such as simultaneous equations often went astray, and obtaining a positive value for the Y- intercept or a large value for the gradient evidently did not cause concern. It would seem that a significant minority did not connect this part with earlier parts of the question..	condone values from table; condone slips eg in reading from graph
		iii	h = their$a \times 10^{\text{their}bt}$ or $h = 10^{\text{their } \log a + \text{their}bt}$	M1		if B1M1M0, then SC1 for $\log h = \log a$ + theirbt isw if both values in the acceptable range for A1
		iii	$0.028 \le b \le 0.032$ and $0.5 \le a \le 0.603$ or $-0.3 \le \log a \le -0.22$	A1		
		iv	$a10^{60b} - a10^{50b}$ their values for a and b	M1	or $10^{\log a + b \times 60} - 10^{\log a + b \times 50}$ or their values for log$a$ and b **Examiner's Comments** t = 70, 10 and 55 were all seen, but many candidates used t = 60 successfully with their model, and then subtracted either 15.9 or f(50) to earn both marks. Unfortunately a few candidates stopped at f(60) lost both marks.	condone 15.9 as second term may follow starting with $\log h = \log a$ + theirbt
		iv	8.0 to 26.1 inclusive	A1		NB A0 for estimate without clear valid method using model; both marks available even if a or b or both are outside range in (iii)
		v	comment on the **continuing reduction** in thickness **and** its consequences	B1	eg in long term, it predicts that reduction in thickness will continue to increase, even when the glacier has completely melted **Examiner's Comments** Many candidates wrote sensible and worthy responses to this question. Unfortunately, many of them failed to score, in spite of their likely truth, as they were vague or missed the point. Candidates were expected to comment on the model continuing to predict an ever increasing rate of reduction in the thickness of the	

					ice, in spite of the fact that at some point all the ice will have melted.	
			Total	13		
5			$(x+1)\log 3 = 2x\log 5$ oe	M1	or $x + 1 = 2x\log_3 5$ or $(x+1)\log_5 3 = 2x$	allow recovery from omission of brackets in later working
			$\log 3 = x(2\log 5 - \log 3)$ oe	A1	$x(1 - 2\log_3 5) = -1$ oe or $x(2 - \log_5 3) = \log_5 3$ oe	NB $0.477121254 = 0.920818754x - 1.929947041x = -1$ $1.317393806x = 0.682606194$..
			$\dfrac{\log 3}{2\log 5 - \log 3}$ oe	A1	$\dfrac{1}{2\log_3 5 - 1}$ oe or $\dfrac{\log_5 3}{2 - \log_5 3}$ oe	
			0.518 cao	A1	**Examiner's Comments** Most candidates understood the initial step, but many omitted the brackets and never recovered. Many of those who did earn the first mark often made errors in manipulating the equation, and scored no further marks. The best candidates usually went on to score 4/4.	answer only does not score
			Total	4		
6			$m = 3$ seen	B1		
			$\log y = m\log x + 2$ or $\log y = m\log x + \log 100$	M1	or $\log y - 8 = m(\log x - 2)$	condone lack of base; "$c = 2$" is insufficient
			$\log y = \log x^3 + 2$ or $\log y = \log x^3 + \log 100$ or better	M1	or $10^{\log y} = 10^{3\log x + 2}$ or $10^{3\log x + \log 100}$ or better	condone lack of base, but not bases other than 10 unless fully recovered
			$y = 100x^3$ or $y = 10^{3\log x + 2}$ or $y = 10^{\log x^3 + 2}$ www isw	A1	$y = 10^{3\log x + \log 100}$ or $y = 10^{\log x^3 + \log 100}$	

					Examiner's Comments A minority of candidates found this question straightforward and produced fully correct solutions. However, the majority struggled or failed to give sufficient detail of their working to earn full credit. A good number found the gradient of the line as 3. Some used $\frac{\log 6}{\log 2}$, indicating the common misconception of the model. log$y$ = 3log$x$ + log 2 was very common as a second statement. Those who earned the second mark very often lost the thirdfor statements such as y = 3x + 2 (removing all the "logs") or $y = x^3 + 100$, without log$y$ = logx^3 + 2, or equivalent, having been seen. It is important that eachstep should be shown as correct final answers were often seen following incorrect working, which of course do not score. A few candidates knew that the final model was of the form $y = ax^b$ and also demonstrated that b was the gradient and a was 10$^{\text{(the intercept)}}$, producing the correct equation relating y and x. Many of these candidates would have done better to re-read the question as most of them omitted to state the equation relating logy and logx, which was one of the demands of the question.	
			Total	4		
7		i	both curves with positive gradients in 1st and 2nd quadrants; ignore labels for this mark	M1	do not award if clearly not exponential shape; condone touching negative x-axis but not crossing it	consider each curve independently; ignore scales and points apart from (0, 1)
		i	both through (0, 1)	A1		allow if indicated in table of values or commentary if notmarked on graph
		i	$y = 3^{2x}$ above $y = 3^x$ in first quadrant and below it in second	A1	must be clearly labelled, **A0** if wrongly attributed or if coincide for negative x from (0, 1) **Examiner's Comments**	if **M0** allow **SC1** for one graph fully correct

					A small number of candidates drew two curves of totally different shapes, which was surprising, but most knew the correct shape and although many sketches were sloppily presented, and marks were lost through omitting to identify (0, 1) or by allowing the curves to coalesce through the second quadrant.	
		ii	$x = 3$	B1	**B0** if wrongly attributed	
		ii	$3^x = 27$	B1	**B0** if wrongly attributed **Examiner's Comments** This was very well done. Nearly all candidates correctly found $x = 3$; a few thenevaluated 3^3 as 6, 9 or 81.	allow $3^3 = 27$ with $x = 3$ stated
			Total	5		
8		i	$\log_{10}y = \log_{10}a + bt$ www	B1	**B0** for just $\log_{10}y = \log_{10}a + bt\log_{10}10$	allow omission of base throughout question
		i	gradient is b, intercept is $\log_{10}a$ cao	B2	**B1** for one correct; award independently of their equation; must be stated – linking by arrows etc is insufficient; condone $m = b$ and $c = \log a$ **Examiner's Comments** Many scored full marks in this part, but of those who derived the equation, a significant minority did so incorrectly, thus losing the first mark. "bt" was sometimes quoted as the gradient, and "a = intercept" was a common error. Some candidates failed to state the gradient or the intercept, simply drawing lines to their equation or linking with $y = mx + c$. This is insufficient.	ignore t-intercept is $\frac{-\log_{10} a}{b}$ **B0** for gradient is bt
		ii	1.58, 1.8[0], 1.98, 2.37, 2.68	B1	allow values which round to these numbers to 2 dp;	all values must be correct
		ii	all values correct and all plotted accurately	B1	within tolerance on overlay;	

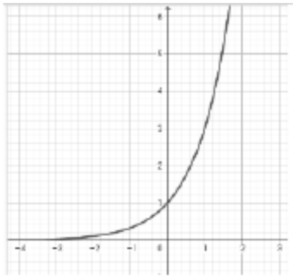
		ii	ruled line of best fit for at least $1 \le t \le 10$	B1	within tolerance on overlay: must not cut red or green line; line between (1, 0.6) and (1, 1.05) at lower limit and between (10, 2.3) and (10, 2.75) at upper limit;	use ruler tool to check if line is ruled where necessary; tolerance: one small square horizontally at each end; not dependent on correct plots
		ii	evalution of $\dfrac{\log y_2 - \log y_1}{t_2 - t_1}$ or substitution of (t_1, logy_1) and (t_2, log y_2) in log $y = bt + \log a$ to obtain a numerical value for the gradient	M1	(t_1, log y_1) and (t_2, log y_2) are points on their line	condone use of values from table
		ii	$0.14 \leqslant b \leqslant 0.24$	A1	gradient must be identified as *b* for **A1**	
		ii	$2.5 \leqslant a \leqslant 6.3$	B1	must be identified as *a*; not from wrong working	
		ii	y = their $a \times 10^{\text{their } b \times t}$ or $y = 10^{\text{their} bt + \text{their log} a}$ or $10^{\text{their log } a} \times 10^{\text{their} b \times t}$ oe	M1		if **M0A0B0M0** allow **SC3** for substitution directly into given formula to obtain $y = a10^{bt}$ with *a* **and** *b* in acceptable range
		ii	*a* and *b* or log *a* and *b* both in acceptable range	A1	$0.4 \le \log a \le 0.8$ **Examiner's Comments** Most completed the table successfully, and went on to plot the points and draw a suitable line of best fit. A few lost an easy first mark through poor calculator skills (2.34 instead of 2.37 was quite common) and some rounded to 1 decimal place. A few candidates drew a curve of best fit, or failed to use a ruler. Most were able to find the gradient of the line for an easy mark, but many failed to link this to *b*. Similarly, the instruction to find the value of a was often disregarded. Surprisingly, many candidates simply stopped when they had found a and *b*, thus losing the last two marks.	
		iii	260 **or** 261	B1	**B0** for non-integer answer **Examiner's Comments**	

					The majority of candidates successfully obtained the correct value, but a significant minority lost an easy mark by failing to give the answer in context as an integer.	
			Total	12		
9		i	$\log_a 1 = 0$ soi or $3m\log_a a$ or $\log_a a^{-3m}$ seen	M1	do not condone $3m\log a$	do not allow MR for $(\log_a a^m)^3$
					Examiner's Comments	
		i	$-3m$ cao	A1	Most candidates achieved a method mark from $\log_a 1 = 0$, but were often unable to resolve the second term. Surprisingly, a few candidates dealt successfully with $\log_a (a^m)^3$, but not with the first term.	
		ii	$(2x + 1)\log_3 3 = \log_3 1000$ or $2x + 1 = \log_3 1000$ oe	M1	Or $(2x + 1)\log_{10} 3 = \log_{10} 1000$ [= 3]	condone omission of brackets; allow omission of base 10 or consistent use of other base
		ii	$[x =]\dfrac{\log_3 1000 - 1}{2}$ oe	M1	or $[x =]\dfrac{\frac{3}{\log_{10} 3} - 1}{2}$ oe not from wrong working	allow one sign error and / or omission of brackets allow recovery from bracket error for **A1**
		ii	2.64 cao; mark the final answer	A1	**Examiner's Comments** This was done very well indeed. A small number of candidates slipped up in making x the subject, and a few lost the final mark by giving the answer correct to three decimal places.	0 if unsupported or for answer obtained by trial and error on $3^{2x+1} = 1000$
			Total	5		
10		a	$\log_{10} N = \log_{10} A + kt\log_{10} 2$	M1(AO1.1) E1(AO1.2)		

			Equation above is of the form $y = mx + c$ [with $\log_{10} N$ as y and t as x] Gradient = $k \log_{10} 2$ Intercept = $\log_{10} A$	A1(AO2.2a) A1(AO2.2a) [4]		
		b	$k \log_{10} 2 = 0.2 \Rightarrow k = 0.66[438]$ $\log_{10} A = 2 \Rightarrow A = 100$	B1(AO1.1) B1(AO1.1) [2]		
		c	$N = 100 \times 2^{0.66\ldots \times 24} = 6\ 300\ 000$ FT their A, k	B1(AO3.4) [1]	Answer in range 5 860 000 to 6 400 000	
		d	E.g. the piece of bread may not be sufficient to support the number of bacteria	E1(AO3.5b) [1]	OR bacterial growth may obey different rules for large values of t	
			Total	8		
11		a	$\log_3 x^2 a$	B1(AO1.1)		

				[1]		
		b	$x^2a = 3^2$ $x = [\pm]\frac{3}{\sqrt{a}}$ oe Disregard $x = -\frac{3}{\sqrt{a}}$ as x cannot be negative	M1(AO1.1) A1(AO1.1) A1(AO2.1) [3]	Must be clear that the negative root has been considered and disregarded	
			Total	4		
12		a	$c = 4.45$	B1(AO3.3) [1]		
		b	$\log_{10}y = -0.37 \log_{10}t + 4.45$ $y = 10^{-0.37 \log_{10}t + 4.45}$ $\log_{10}t^{-0.37}$ seen $10^{4.45} \times t^{-0.37}$ 21183.829.. ≈ 28 200 so $y \approx 28\ 200\ t^{-0.37}$	M1(AO2.1) M1(AO1.1) A1(AO1.1) [3]	may be awarded after combining logarithms	

					AG	
		c	18 781 is close to 18776	B1(AO3.5a) [1]	BC	NB $t = 3$	
		d	**A** 12 507 or 12 508	B1(AO3.4) [1]	BC to 3, 4 or 5 s.f.	
		d	**B** 8986	B1(AO3.4) [1]	BC	
		e	Answer to A interpolation so more likely to be reliable Answer to B extrapolation beyond 2015 so unreliable	B1(AO3.5a) B1(AO3.5b) [2]		
			Total	9		
13		a	4	B1(AO1.2) [1]	<u>Examiner's Comments</u> Many candidates gave the correct answer to this part. However, some candidates omitted this, and some others gave answers in terms of *a*.	
		b	−1	B1(AO1.1) [1]		

					Examiner's Comments This part was sometimes omitted, and some candidates gave answers involving *a*.	
			Total	2		
14		a	(i) 	B1 (AO 1.2) [1]	correct shape in both quadrants / condone touching the *x*-axis, but not cutting it Examiner's Comments Sketches should make clear that the function tends towards $y = 0$ but does not cut the *x*-axis	
			(ii) (0, 1)	B1 (AO 1.1) [1]	do not allow just $y = 1$ Examiner's Comments A clear indication of the coordinates (0,1) was required and not just $y = 1$.	
		b	$(f(x) =)\log_3 x$	B1 (AO 1.1) [1]	allow eg $\frac{\log x}{\log 3}$ Examiner's Comments There was an expectation that candidates would use base 3	

					logarithms, but a variety of correct functions were given and gained credit.	
			Total	3		
15		a	$C = 2$ $A = 62$ $B = 10$	B1 (AO 3.3) B1 (AO 3.3) B1 (AO 1.1) [3]	since max when $t = 2$ since max when $(t - 2)^2 = 0$ from substitution of 22, 62 and 2 **Examiner's Comments** Candidates who did well in this question recognised that the maximum value of p has to be 62, and that this occurs when $t = 2$, thus obtaining A and C. The value of B soon follows. This who did less well wrote down three equations in three variables and often went astray.	
		b	substitution of 0.75 in $p = 62 - 10(t - 2)^2$ 46	M1 (AO 3.4) A1 (AO 1.1) [2]	FT *their* 2, 62, 10 allow 46.375 rounded to 2 or more sf **Examiner's Comments** Candidates who did well made the correct substitution in their formula. Candidates who did less well substituted $t = 45$.	

		c	*their* $62 - 10(t-2)^2 = 0$ [t =] 4 hours 29 minutes or 4 hours 30 minutes	M1 (AO 3.4) A1 (AO 2.4) [2]	or ≥ or > 0 for **M1** NB $t = 2 + \sqrt{6.2}$ allow 4.49 or 4.5 [hours] Examiner's Comments Candidates who did well understood that the value of p couldn't be negative and obtained a value for t accordingly. Candidates who did less well worked with < instead of = or >.	
		d	substitution of t = 1, 3 and 5 awrt 59.4 ≈ 59 awrt 83.8 ≈ 84 awrt 88.8 ≈ 89	M1 (AO 3.4) A1 (AO 3.5a) [2]	or awrt 59.4, 83.8 and 88.8 found and supporting comment made eg they are approximately the same as the values in the table if **M0** allow **SC1** for two values correctly found and shown to be consistent or supporting comment made Examiner's Comments Candidates who did well made correct substitutions and provided a supporting comment. Candidates who did less well neglected to substitute all three values or made no comment on what their calculations showed.	

		e	$p \to 90$ as $t \to$ large or when $t = 12$ $p = 89.99539\ldots$rounded to 2 or more sf comparison with value of p for $t = 5$ eg model predicts $p = 89$ for $t = 5$ and $p = 90$ for $t = 12$ so not good advice	B1 (AO 3.5a) B1 (AO 3.5a) [2]	or model predicts $p = 90$ for (any) $t \geq 7$ so not good advice | allow equivalent comment on 7 hours work for one extra mark Examiner's Comments Candidates who did well evaluated p at $t = 12$ and either at $t = 5$ or a value in between 5 and 12 and commented appropriately. Candidates who did less well made no supporting calculations but supplied a comment, or vice versa.	
			Total	11		
16		a	$\log_{10}y = \log_{10}A + kx$ Equation is of straight line form '$Y = mX + c$', hence model is consistent with graph	M1 (AO 1.1) E1 (AO 2.4) [2]		
		b	$\log_{10}A = 4.83$ $A = 67600$ $k = \dfrac{4.83 - 4.6}{0 - 7}$	M1 (AO 1.1a) A1 (AO 2.2a) M1 (AO 1.1) A1 (AO	Attempt to find	

			$k = -0.0328\ldots$	2.2a) [4]	gradient Allow – 0.0325 to – 0.033	
		c	$\log_{10} 10000 < 4.83 - 0.0328x$ $x > 25.3$ 2036	M1 (AO 3.4) A1 (AO 1.1) A1 (AO 3.2a) [3]	Use of their equation	
		d	Not reliable, as extrapolation may not be valid	A1 (AO 3.5b) [3]	Accept any reasons for 'not reliable' that refer to possible future changes of circumstances, oe	
			Total	10		
17			$5 \log_a x$ or $\log_a x^{-1}$ oe soi $6 \log_a x$	M1 (AO1.1) A1 (AO1.1) [2]		
			Total	2		
18		a	5 million	B1(AO 3.4) [1]		

		b	The population is growing At a rate proportional to the population	B1(AO 3.2a) B1(AO 3.2a) [2]	**or** at 2% a year **or** by the same percentage each year	
		c	DR $5 \times 1.02^n = 10 \Rightarrow 1.02^n = 2$ $n = \frac{\log 2}{\log 1.02} = 35.002...$ 2035	M1(AO 3.4) M1(AO 1.1a) A1(AO 3.2a) [2]	oe	
		d	$\log_{10} P = \log_{10} 5 + n \log_{10} 1.02$ Of form $y = mx + c$ [with $\log_{10} P$ as y and n as x]	M1(AO 1.1) E1(AO 2.4) [2]	oe with clear link to gradient and intercept	
			Total	8		