Question	Scheme	Marks	AOs
1	$\dfrac{9^{x-1}}{3^{y+2}}=81\Rightarrow\dfrac{3^{2x-2}}{3^{y+2}}=3^4$ or $\dfrac{9^{x-1}}{3^{y+2}}=81\Rightarrow\dfrac{9^{x-1}}{9^{\frac{1}{2}(y+2)}}=9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4\Rightarrow y=$ or $\Rightarrow x-1-\dfrac{1}{2}y-1=2\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\dfrac{9^{x-1}}{3^{y+2}}\right)=\log_3 81$	M1	1.1b
	$\Rightarrow(x-1)\log_3\left(9^{x-1}\right)-(y+2)\log_3\left(3^{y+2}\right)=4$ $\Rightarrow 2(x-1)-y-2=4\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3 marks)	

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1}=3^{2x-1}$ and $9^{x-1}=3^{3x-3}$.
Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach *y* in terms of *x*. Condone slips in their rearrangement.

A1: $y=2x-8$

Question	Scheme	Marks	AOs
2 (a)	$35\ (\text{km}^2)$	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14}\ln\left(\frac{20}{45}\right) = \cdots -0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
		(6 marks)	

Notes

(a)

B1: Uses the equation of the model to find that $35\ (\text{km}^2)$ of the reserve was covered on 1st January 2005. Do not accept eg. $35\ \text{m}^2$

(b)

M1: Sets their $60 = 80 - 45e^{14c} \Rightarrow Ae^{14c} = B$

A1: $45e^{14c} = 20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and $\ln x$ are inverse functions and proceeds to a value for c.

A1: Gives a complete equation for the model $A = 80 - 45e^{-0.0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

Question	Scheme	Marks	AOs
3(a)	$(k=)\ 0.8$	B1	1.1b
		(1)	
(b)	$1=0.8+1.4e^{-0.5t} \Rightarrow 1.4e^{-0.5t}=0.2$	M1	3.1b
	$-0.5t=\ln\left(\frac{0.2}{1.4}\right) \Rightarrow t=...$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{dP}{dt}=\right)\ -0.7e^{-0.5t}$ $\left(\frac{dP}{dt}\right)_{t=2}=-0.7e^{-0.5\times 2}$	M1	3.1b
	= awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)
B1: Completes the equation for the model by obtaining $(k=)$ 0.8 or equivalent.

(b) ***Be aware this could be solved entirely using a calculator which is not acceptable***

M1: For using the model with $P=1$ and their value for k from (a) and proceeding to $Ae^{\pm 0.5t}=B$. Condone if A or B are negative for this mark.

M1: Uses correct log work to solve an equation of the form $Ae^{\pm 0.5t}=B$ leading to a value for t. They cannot proceed directly to awrt 3.9 without some intermediate working seen.
Eg $t=2\ln 7$ or $-2\ln\left(\frac{1}{7}\right)$ is acceptable.
Also allow $1.4e^{-0.5t}=0.2 \Rightarrow -0.5t=-1.9459... \Rightarrow t=...$
This cannot be scored from an unsolvable equation (eg when their $k \geq 1$ so that $e^{\pm 0.5t} \leq 0$).

A1: Accept awrt 3.9 minutes or t = awrt 3.9 with correct working seen.
eg $1.4e^{-0.5t}=0.2 \Rightarrow t=3.9$ would be M1M0A0

(c) ***Be aware this can be solved entirely using a calculator which is not acceptable***

M1: Links rate of change to gradient and differentiates to obtain an expression of the form $Ae^{-0.5t}$ and substitutes $t=2$. Do not accept $Ate^{-0.5t}$ as the derivative.
Beware that substituting $t=2$ and proceeding from e^{-1} to e^{-2} is M0A0

A1: Obtains awrt 0.258 with differentiation seen. (Units not required) Condone awrt -0.258
Awrt ± 0.258 with no working is M0A0. Isw after a correct answer is seen.

(Ignore in (c) any spurious notation on the LHS when differentiating such as $P=....$ or $\frac{dy}{dx}=...$)

Question	Scheme	Marks	AOs
4(a)	$A = 1000$	B1	3.4
	$2000 = 1000e^{5k}$ or $e^{5k} = 2$	M1	1.1b
	$e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = ...$	M1	2.1
	$N = 1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt} = 1000 \times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)e^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8} = 1000 \times 0.139e^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4\times\left(\frac{1}{5}\ln 2\right)T} = 1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4 \times "0.339"T = \ln 2 + "0.339"t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N = 1000e^{...t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N = 2000, t = 5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $ae^{5k} = b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N = 1000e^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N = 1000 \times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates αe^{kt} to βe^{kt} and substitutes $t = 8$ (Condone $\alpha = \beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

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Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000e^{0.139t}$, and then writes at $t = 8$ $\frac{dN}{dt} =$ awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500e^{1.4\times"0.139"t} = 1000e^{"0.139"t}$

If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0

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Question	Scheme	Marks	AOs
5 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
			(8 marks)
Notes:			

(a) May be seen in the question so watch out.

B1: Accept 265 thousand or 265 000 or equivalent such as 265 k but not just 265.

(b)

M1: Differentiates to a form $ke^{0.05t}, k > 0, k \neq 220$. Do not be too concerned about the lhs.

M1: Substitutes $t = 10$ into a changed function that was formed from an attempt at differentiation.

The left hand side must have implied differentiation. E.g. Rate = , $N', \frac{dN_b}{dt}, \frac{dN}{dt}$ or even $\frac{dy}{dx}$

A1*: Full and complete proof that requires

- some correct lhs seen at some point. E.g. "Rate = , " $\frac{dN_b}{dt}, \frac{dN}{dt}$ but condone N'.
- an intermediate line/answer of either $11e^{0.05\times10}$ or awrt 18.1 before a minimal conclusion which must be referencing the 18 000 or 18 thousand

(c)

M1: Attempts to set both equations equal to each other and simplify the constant terms.

Look for $220\,e^{0.05t} + 35 = 800\,e^{-0.05t}$ o.e but condone slips

It is also possible to set $\frac{N-45}{220} = \left(e^{0.05t} = \right) \frac{800}{N-10}$ and form an equation in N

A1: Correct quadratic form.

Look for $220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$ or $220\,e^{0.1t} + 35e^{0.05t} - 800 = 0$ but allow with terms in different order such as $220\,e^{0.1t} + 35e^{0.05t} = 800$

FYI the equation in N is $N^2 - 55N - 175550 = 0$

M1: Full attempt to find the value of t (or a constant multiple of t)

This involves the key step of recognising and solving a 3TQ in $e^{0.05t}$ followed by the use of lns.
If the answers to the quadratic just appear (from a calculator) you will need to check.
Accuracy should be to 3sf.
You may see different variables used such as x

$x = e^{0.05t}$, $220\,e^{0.1t} + 35e^{0.05t} = 800 \Rightarrow 220x^2 + 35x = 800 \Rightarrow x = 1.82... \Rightarrow t = 20\ln 1.82...$

Allow use of calculator for solving the quadratic and for $e^{0.05t} = 1.82.. \Rightarrow t = 12.08$

Via the N route it will involve substituting the positive solution to their quadratic into either equation to find a value for t/T using same rules as above.

A1: AWRT 12.08

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Answers with limited or no working in (b) and (c)

(b) A derivative in the correct form must be seen

(c) Candidates who state $45 + 220\,e^{0.05t} = 10 + 800\,e^{-0.05t}$ followed by awrt 12.08 (presumably from using num-solv on their calculators) can score SC 1100. Rubric on the front of the paper states that "Answers without working may not gain full credit" so we demand a method in this part.

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Question	Scheme	Marks	AOs
6(a)	$(A=)\ 55$	B1	3.4
		(1)	
(b)	$\left\{\frac{dH}{dt}=\right\}-ABe^{-Bt}$ or $\left\{\frac{dH}{dt}=\right\}-"55"Be^{-Bt}$	M1	3.1b
	$-B\times"55"=-7.5\Rightarrow B=...\ \left(\frac{3}{22}=\text{awrt } 0.136\right)$	M1	1.1b
	$H=55e^{-0.136t}+30$	A1cso	3.3
		(3)	
			(4 marks)

Notes

(a)

B1: 55 only. Just look for this value e.g. "*A* =" is not required. Ignore any "units" if given e.g. 55 °C

(b)

M1: Differentiates to obtain an expression of the form $\pm ABe^{-Bt}$ which may have their *A* already substituted in so allow for $\pm ABe^{-Bt}$ or $\pm"55"Be^{-Bt}$

M1: Substitutes $t = 0$ and their *A* into their $\frac{dH}{dt}$, sets $=\pm 7.5$ and proceeds to find a value for *B* which may be implied by $\frac{3}{22}$ or awrt 0.136

Their $\frac{dH}{dt}$ must not be *H*. i.e. it must be a "changed" function.

A1cso: Correct **equation** which follows **fully correct work** $H=55e^{-0.136t}+30$ but condone $H=55e^{-\frac{3}{22}t}+30$ The final equation must be correct but you can ignore spurious notation within their solution such as integral signs and "+ *c*" which do not affect their solution.

<u>Marking guidance is as follows for particular cases in (b)</u>

Case 1: $\left\{\frac{dH}{dt}=\right\}-"55"Be^{-Bt}$, $-"55"Be^{-Bt}=7.5\Rightarrow\ B=-0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: it should be – 7.5

Case 2: $\left\{\frac{dH}{dt}=\right\}"55"Be^{-Bt}$, $"55"Be^{-Bt}=-7.5\Rightarrow\ B=-0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 3: $\left\{\frac{dH}{dt}=\right\}"55"Be^{-Bt}$, $"55"Be^{-Bt}=7.5\Rightarrow\ B=0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 4: $\left\{\frac{dH}{dt}=\right\}-"55"Be^{-Bt}$, $"55"B=7.5\Rightarrow\ B=0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A1**

No errors