DIFFERENTIATION

1 The curve C has equation $y = \frac{1}{4x} - \ln x$.

a Find the gradient of C at the point $(1, \frac{1}{4})$. **(3)**

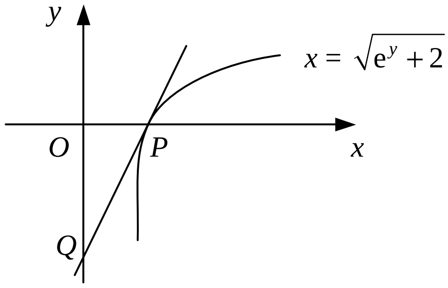
b Find an equation for the normal to C at the point $(1, \frac{1}{4})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

2 A curve has the equation $y = xe^{-2x}$.

a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **(4)**

b Find the exact coordinates of the turning point of the curve and determine its nature. **(4)**

3



The diagram shows the curve $x = \sqrt{e^y + 2}$ which crosses the x-axis at the point P.

a Find the coordinates of P. **(1)**

b Find $\frac{dx}{dy}$ in terms of y. **(2)**

The tangent to the curve at P crosses the y-axis at the point Q.

c Show that the area of triangle OPQ, where O is the origin, is $3\sqrt{3}$. **(5)**

4 A rock contains a radioactive substance which is decaying.

The mass of the rock, m grams, at time t years after initial observation is given by

$$m = 600 + 80e^{-0.004t}.$$

a Find the percentage reduction in the mass of the rock over the first 100 years. **(3)**

b Find the value of t when $m = 640$. **(2)**

c Find the rate at which the mass of the rock will be decreasing when $t = 150$. **(3)**

5 Differentiate with respect to x

a $\sqrt{\sin x + \cos x}$, **(3)**

b $\ln \left(\frac{x-1}{2x+1} \right)$. **(3)**

6 A curve has the equation $y = (2x - 3)^5$.

a Find an equation for the tangent to the curve at the point P $(1, -1)$. **(4)**

Given that the tangent to the curve at the point Q is parallel to the tangent at P,

b find the coordinates of Q. **(3)**

7 A curve has the equation $y = \frac{2}{x^2 - 5}$.

a Find the coordinates of the stationary point of the curve. **(4)**

b Show that the tangent to the curve at the point with x-coordinate 3 has the equation

$$3x + 4y - 11 = 0.$$ **(3)**

8 $$\text{f} : x \to a\text{e}^x + a, \quad x \in \mathbb{R}.$$

Given that a is a positive constant,

a sketch the graph of $y = \text{f}(x)$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. **(2)**

b Find the inverse function f^{-1} in the form $\text{f}^{-1} : x \to \ldots$ and state its domain. **(4)**

c Find an equation for the tangent to the curve $y = \text{f}(x)$ at the point on the curve with x-coordinate 1. **(4)**

9 **a** Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{\text{d}}{\text{d}x}(\cot x) = -\text{cosec}^2\, x.$$ **(4)**

b Show that the curve with equation

$$y = \text{e}^x \cot x$$

has no turning points. **(5)**

10 A curve has the equation $y = (2 + \ln x)^3$.

a Find $\frac{\text{d}y}{\text{d}x}$. **(2)**

b Find, in exact form, the coordinates of the stationary point on the curve. **(3)**

c Show that the tangent to the curve at the point with x-coordinate e passes through the origin. **(3)**

11 $$\text{f} : x \to \ln (9 - x^2), \quad -3 < x < 3.$$

a Find $\text{f}'(x)$. **(2)**

b Find the coordinates of the stationary point of the curve $y = \text{f}(x)$. **(2)**

c Show that the normal to the curve $y = \text{f}(x)$ at the point with x-coordinate 1 has equation

$$y = 4x - 4 + 3\ln 2.$$ **(4)**

12 A botanist is studying the regeneration of an area of moorland following a fire.

The total biomass in the area after t years is denoted by M tonnes and two models are proposed for the growth of M.

Model A is given by

$$M = 900 - \frac{1500}{3t + 2}.$$

Model B is given by

$$M = 900 - \frac{1500}{2 + 5\ln(t + 1)}.$$

For each model, find

a the value of M when $t = 3$, **(2)**

b the rate at which the biomass is increasing when $t = 3$. **(6)**