

DIFFERENTIATION

Answers

1 a (5, 2)

b $\frac{4}{t} = -8 \therefore t = -\frac{1}{2}$

2 a (2, 0)

b $1 + \sin t = \frac{3}{2}, \sin t = \frac{1}{2}, t = \frac{\pi}{6}, \frac{5\pi}{6}$

$2 \cos t = -\sqrt{3}, \cos t = -\frac{\sqrt{3}}{2}, t = \frac{5\pi}{6}, \frac{7\pi}{6}$

$\therefore t = \frac{5\pi}{6}$

3 a $t = \frac{x}{3} \therefore y = \left(\frac{x}{3}\right)^2$

$y = \frac{1}{9}x^2$

b $t = \frac{x}{2} \therefore y = \frac{1}{\left(\frac{x}{2}\right)^2}$

$y = \frac{2}{x}$

c $x^2 = t^6, y^3 = 8t^6$

$\therefore y^3 = 8x^2$

d $t = 4 - y$

$\therefore x = 1 - (4 - y)^2$

e $t = \frac{1}{2}(x + 1)$

$\therefore y = \frac{2}{\frac{1}{4}(x+1)^2}$

$y = \frac{8}{(x+1)^2}$

f $t = \frac{1}{x} + 1$

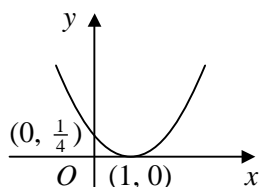
$\therefore y = \frac{1}{2 - \left(\frac{1}{x} + 1\right)} = \frac{1}{1 - \frac{1}{x}}$

$y = \frac{x}{x-1}$

4 a $t = \frac{1}{2}(x - 1)$

$\therefore y = \frac{1}{4}(x - 1)^2$

b



5 a $\cos^2 \theta + \sin^2 \theta = 1$

$\therefore x^2 + y^2 = 1$

b $\cos 2\theta = 1 - 2\sin^2 \theta$

$\therefore y = 1 - 2x^2$

c $\cos \theta = \frac{x-3}{2}, \sin \theta = \frac{y-1}{2}$

$\cos^2 \theta + \sin^2 \theta = 1$

$\therefore \left(\frac{x-3}{2}\right)^2 + \left(\frac{y-1}{2}\right)^2 = 1$

$(x-3)^2 + (y-1)^2 = 4$

d $\sec \theta = \frac{x}{2}, \tan \theta = \frac{y}{4}$

$1 + \tan^2 \theta = \sec^2 \theta$

$\therefore 1 + \left(\frac{y}{4}\right)^2 = \left(\frac{x}{2}\right)^2$

$16 + y^2 = 4x^2$

$y^2 = 4x^2 - 16$

e $\sin 2\theta = 2 \sin \theta \cos \theta$

$\therefore y = 4 \sin^2 \theta \cos^2 \theta$

$y = 4 \sin^2 \theta (1 - \sin^2 \theta)$

$y = 4x^2(1 - x^2)$

f $\sec \theta = \frac{1}{x}$

$1 + \tan^2 \theta = \sec^2 \theta$

$\therefore 1 + y = \left(\frac{1}{x}\right)^2$

$y = \frac{1}{x^2} - 1$

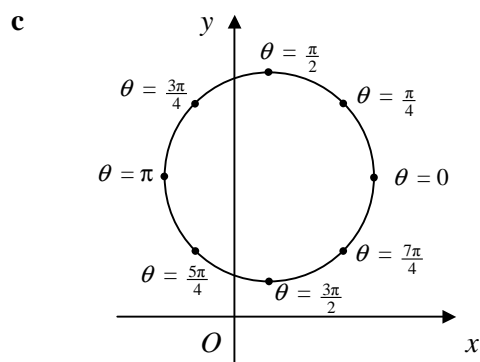
6 a $\cos \theta = \frac{x-1}{3}$, $\sin \theta = \frac{y-4}{3}$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\left(\frac{x-1}{3}\right)^2 + \left(\frac{y-4}{3}\right)^2 = 1$$

$$(x-1)^2 + (y-4)^2 = 9$$

b centre (1, 4) radius 3



7 a $x = 5 \cos \theta$, $y = 5 \sin \theta$, $0 \leq \theta < 2\pi$

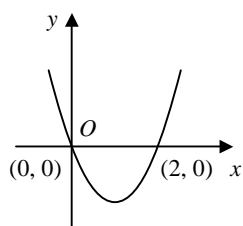
b $x = 6 + 2 \cos \theta$, $y = -1 + 2 \sin \theta$, $0 \leq \theta < 2\pi$

c $x = a + r \cos \theta$, $y = b + r \sin \theta$, $0 \leq \theta < 2\pi$

8 a $t = \frac{x}{2}$

$$\therefore y = 4\left(\frac{x}{2}\right)\left(\frac{x}{2} - 1\right)$$

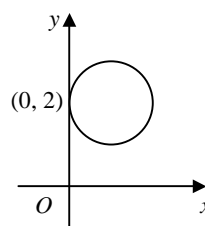
$$y = x(x-2)$$



b $\sin \theta = 1 - x$, $\cos \theta = 2 - y$

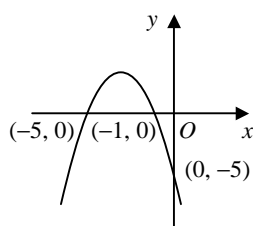
$$\cos^2 \theta + \sin^2 \theta = 1 \quad \therefore (2-y)^2 + (1-x)^2 = 1$$

$$\text{or } (x-1)^2 + (y-2)^2 = 1$$



c $t = x + 3$

$$\therefore y = 4 - (x+3)^2$$



d $t = x - 1$

$$\therefore y = \frac{2}{x-1}$$

