DIFFERENTIATION

1 A curve is given by the parametric equations

$$x = t^2 + 1, \quad y = \frac{4}{t}.$$

a Write down the coordinates of the point on the curve where $t = 2$.

b Find the value of t at the point on the curve with coordinates $(\frac{5}{4}, -8)$.

2 A curve is given by the parametric equations

$$x = 1 + \sin t, \quad y = 2\cos t, \quad 0 \leq t < 2\pi.$$

a Write down the coordinates of the point on the curve where $t = \frac{\pi}{2}$.

b Find the value of t at the point on the curve with coordinates $(\frac{3}{2}, -\sqrt{3})$.

3 Find a cartesian equation for each curve, given its parametric equations.

a $x = 3t, \quad y = t^2$ **b** $x = 2t, \quad y = \frac{1}{t}$ **c** $x = t^3, \quad y = 2t^2$

d $x = 1 - t^2, \quad y = 4 - t$ **e** $x = 2t - 1, \quad y = \frac{2}{t^2}$ **f** $x = \frac{1}{t-1}, \quad y = \frac{1}{2-t}$

4 A curve has parametric equations

$$x = 2t + 1, \quad y = t^2.$$

a Find a cartesian equation for the curve.

b Hence, sketch the curve.

5 Find a cartesian equation for each curve, given its parametric equations.

a $x = \cos\theta, \quad y = \sin\theta$ **b** $x = \sin\theta, \quad y = \cos 2\theta$ **c** $x = 3 + 2\cos\theta, \quad y = 1 + 2\sin\theta$

d $x = 2\sec\theta, \quad y = 4\tan\theta$ **e** $x = \sin\theta, \quad y = \sin^2 2\theta$ **f** $x = \cos\theta, \quad y = \tan^2\theta$

6 A circle has parametric equations

$$x = 1 + 3\cos\theta, \quad y = 4 + 3\sin\theta, \quad 0 \leq \theta < 2\pi.$$

a Find a cartesian equation for the circle.

b Write down the coordinates of the centre and the radius of the circle.

c Sketch the circle and label the points on the circle where θ takes each of the following values:

$$0, \ \frac{\pi}{4}, \ \frac{\pi}{2}, \ \frac{3\pi}{4}, \ \pi, \ \frac{5\pi}{4}, \ \frac{3\pi}{2}, \ \frac{7\pi}{4}.$$

7 Write down parametric equations for a circle

a centre (0, 0), radius 5,

b centre (6, –1), radius 2,

c centre (a, b), radius r, where a, b and r are constants and $r > 0$.

8 For each curve given by parametric equations, find a cartesian equation and hence, sketch the curve, showing the coordinates of any points where it meets the coordinate axes.

a $x = 2t, \quad y = 4t(t - 1)$ **b** $x = 1 - \sin\theta, \quad y = 2 - \cos\theta, \quad 0 \leq \theta < 2\pi$

c $x = t - 3, \quad y = 4 - t^2$ **d** $x = t + 1, \quad y = \frac{2}{t}$