

## DIFFERENTIATION

## Answers

- 1**    **a**  $= 5(x+3)^4$       **b**  $= 3(2x-1)^2 \times 2$   
 $= 6(2x-1)^2$       **c**  $= 7(8-x)^6 \times (-1)$   
 $= -7(8-x)^6$       **d**  $= 12(3x+4)^5 \times 3$   
 $= 36(3x+4)^5$
- e**  $= 4(6-5x)^3 \times (-5)$     **f**  $= -(x-2)^{-2}$       **g**  $= -12(2x+3)^{-4} \times 2$     **h**  $= -2(7-3x)^{-3} \times (-3)$   
 $= -20(6-5x)^3$        $= -24(2x+3)^{-4}$       **h**  $= 6(7-3x)^{-3}$
- 2**    **a**  $= 6e^{3t}$       **b**  $= \frac{1}{2}(4t-1)^{-\frac{1}{2}} \times 4$     **c**  $= \frac{5}{t}$       **d**  $= \frac{3}{2}(8-3t)^{\frac{1}{2}} \times (-3)$   
 $= 2(4t-1)^{-\frac{1}{2}}$        $= -\frac{9}{2}(8-3t)^{\frac{1}{2}}$
- e**  $= \frac{3}{6t+1} \times 6$       **f**  $= \frac{1}{2}e^{5t+4} \times 5$       **g**  $= \frac{d}{dx} [6(2t-5)^{-\frac{1}{3}}]$     **h**  $= \frac{2}{3-\frac{1}{4}t} \times (-\frac{1}{4})$   
 $= \frac{18}{6t+1}$        $= \frac{5}{2}e^{5t+4}$        $= -2(2t-5)^{-\frac{4}{3}} \times 2$        $= \frac{2}{t-12}$   
 $= -4(2t-5)^{-\frac{4}{3}}$
- 3**    **a**  $\frac{dy}{dx} = 4(3x-1)^3 \times 3$       **b**  $\frac{dy}{dx} = \frac{4}{1+2x} \times 2$       **c**  $\frac{dy}{dx} = \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times (-2)$   
 $= 12(3x-1)^3$        $= 8(1+2x)^{-1}$        $= -(5-2x)^{-\frac{1}{2}}$   
 $\frac{d^2y}{dx^2} = 36(3x-1)^2 \times 3$        $\frac{d^2y}{dx^2} = -8(1+2x)^{-2} \times 2$        $\frac{d^2y}{dx^2} = \frac{1}{2}(5-2x)^{-\frac{3}{2}} \times (-2)$   
 $= 108(3x-1)^2$        $= \frac{-16}{(1+2x)^2}$        $= -(5-2x)^{-\frac{3}{2}}$
- 4**    **a**  $f'(x) = 2x - \frac{6}{x}$       **b**  $f'(x) = 2 - e^{x-2}$   
 $f'(3) = 6 - 2 = 4$        $f'(2) = 2 - 1 = 1$
- c**  $f'(x) = 4(2-5x)^3 \times (-5) = -20(2-5x)^3$     **d**  $f'(x) = -4(x+5)^{-2}$   
 $f'(\frac{1}{2}) = -20 \times (-\frac{1}{8}) = \frac{5}{2}$        $f'(-1) = -4 \times \frac{1}{16} = -\frac{1}{4}$
- 5**    **a**  $f'(x) = 2(3x+15)^{-\frac{1}{2}} \times 3 = 2$       **b**  $f'(x) = 2x - \frac{1}{x-2} = 5$   
 $\frac{6}{\sqrt{3x+15}} = 2$        $2x(x-2) - 1 = 5(x-2)$   
 $\sqrt{3x+15} = 3$        $2x^2 - 9x + 9 = 0$   
 $3x + 15 = 9$        $(2x-3)(x-3) = 0$   
 $x = -2$       for real  $f(x)$ ,  $x > 2 \therefore x = 3$
- 6**    **a**  $= 3(x^2-4)^2 \times 2x$     **b**  $= 12(3x^2+1)^5 \times 6x$     **c**  $= \frac{1}{3+2x^2} \times 4x$       **d**  $= \frac{d}{dx} [(4-x^2)^3]$   
 $= 6x(x^2-4)^2$        $= 72x(3x^2+1)^5$        $= \frac{4x}{3+2x^2}$        $= 3(4-x^2)^2 \times (-2x)$   
 $= -6x(4-x^2)^2$
- e**  $= \frac{d}{dx} [(\frac{1}{2}x^4+3)^8]$     **f**  $= \frac{d}{dx} [(3-x^2)^{-\frac{1}{2}}]$     **g**  $= 7e^{x^2} \times 2x$       **h**  $= 4(1-5x+x^3)^3 \times (-5+3x^2)$   
 $= 8(\frac{1}{2}x^4+3)^7 \times 2x^3$        $= -\frac{1}{2}(3-x^2)^{-\frac{3}{2}} \times (-2x)$        $= 14xe^{x^2}$        $= 4(3x^2-5)(1-5x+x^3)^3$   
 $= 16x^3(\frac{1}{2}x^4+3)^7$        $= x(3-x^2)^{-\frac{3}{2}}$

$$\mathbf{i} = \frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}}) \quad \mathbf{j} = 7(e^{4x} + 2)^6 \times 4e^{4x} \quad \mathbf{k} = -(5+4\sqrt{x})^{-2} \times 2x^{-\frac{1}{2}} \quad \mathbf{l} = 5(\frac{2}{x}-x)^4 \times (-2x^{-2}-1)$$

$$= \frac{3}{2x-8\sqrt{x}} \quad = 28e^{4x}(e^{4x} + 2)^6 \quad = \frac{-2}{\sqrt{x}(5+4\sqrt{x})^2} \quad = -5(\frac{2}{x^2} + 1)(\frac{2}{x} - x)^4$$

**7 a**  $\frac{dy}{dx} = 5(2x-3)^4 \times 2$       **b**  $\frac{dy}{dx} = 3(x^2-4)^2 \times 2x$       **c**  $\frac{dy}{dx} = 8 - 2e^{2x}$   
 SP:  $10(2x-3)^4 = 0$       SP:  $6x(x^2-4)^2 = 0$       SP:  $8 - 2e^{2x} = 0$   
 $x = \frac{3}{2}$        $x = 0$  or  $x^2 = 4$        $e^{2x} = 4$   
 $\therefore (\frac{3}{2}, 0)$        $x = 0, \pm 2$        $x = \frac{1}{2} \ln 4 = \ln 2$   
 $\therefore (-2, 0), (0, -64), (2, 0)$        $\therefore (\ln 2, 8 \ln 2 - 4)$

**d**  $\frac{dy}{dx} = \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \times 4x$       **e**  $\frac{dy}{dx} = \frac{2}{x-x^2} \times (1-2x)$       **f**  $\frac{dy}{dx} = 4 - (x-3)^{-2}$   
 SP:  $\frac{2x}{\sqrt{1+2x^2}} = 0$       SP:  $\frac{2(1-2x)}{x-x^2} = 0$       SP:  $4 - \frac{1}{(x-3)^2} = 0$   
 $x = 0$        $x = \frac{1}{2}$        $(x-3)^2 = \frac{1}{4}, x-3 = \pm \frac{1}{2}$   
 $\therefore (0, 1)$        $\therefore (\frac{1}{2}, -4 \ln 2)$        $x = \frac{5}{2}, \frac{7}{2}$   
 $\therefore (\frac{5}{2}, 8), (\frac{7}{2}, 16)$

**8 a**  $x = 2 \therefore y = 1$       **b**  $x = 0 \therefore y = 2$   
 $\frac{dy}{dx} = 4(3x-7)^3 \times 3 = 12(3x-7)^3$        $\frac{dy}{dx} = \frac{1}{1+4x} \times 4 = \frac{4}{1+4x}$   
 grad = -12      grad = 4  
 $\therefore y - 1 = -12(x - 2)$        $\therefore y = 4x + 2$   
 [  $y = 25 - 12x$  ]

**c**  $x = 1 \therefore y = 3$       **d**  $x = \frac{1}{4} \therefore y = \frac{1}{2}$   
 $\frac{dy}{dx} = -9(x^2+2)^{-2} \times 2x = -18x(x^2+2)^{-2}$        $\frac{dy}{dx} = \frac{1}{2}(5x-1)^{-\frac{1}{2}} \times 5 = \frac{5}{2}(5x-1)^{-\frac{1}{2}}$   
 grad = -2      grad = 5  
 $\therefore y - 3 = -2(x - 1)$        $\therefore y - \frac{1}{2} = 5(x - \frac{1}{4})$   
 [  $y = 5 - 2x$  ]      [  $y = 5x - \frac{3}{4}$  ]

**9 a**  $x = -2 \therefore y = -9$       **b**  $x = \frac{1}{2} \therefore y = \frac{1}{8}$   
 $\frac{dy}{dx} = e^{4-x^2} \times (-2x) = -2xe^{4-x^2}$        $\frac{dy}{dx} = 3(1-2x^2)^2 \times (-4x) = -12x(1-2x^2)^2$   
 grad = 4  $\therefore$  grad of normal =  $-\frac{1}{4}$       grad =  $-\frac{3}{2}$   $\therefore$  grad of normal =  $\frac{2}{3}$   
 $\therefore y + 9 = -\frac{1}{4}(x + 2)$        $\therefore y - \frac{1}{8} = \frac{2}{3}(x - \frac{1}{2})$   
 [  $y = -\frac{1}{4}x - \frac{19}{2}$  ]      [  $16x - 24y - 5 = 0$  ]

**c**  $x = 1 \therefore y = \frac{1}{2}$       **d**  $x = 3 \therefore y = 6e$   
 $\frac{dy}{dx} = -(2 - \ln x)^{-2} \times (-\frac{1}{x}) = \frac{1}{x(2 - \ln x)^2}$        $\frac{dy}{dx} = 2e^{\frac{x}{e}}$   
 grad =  $\frac{1}{4}$   $\therefore$  grad of normal = -4      grad =  $2e$   $\therefore$  grad of normal =  $-\frac{1}{2e}$   
 $\therefore y - \frac{1}{2} = -4(x - 1)$        $\therefore y - 6e = -\frac{1}{2e}(x - 3)$   
 [  $y = \frac{9}{2} - 4x$  ]      [  $x + 2ey - 12e^2 - 3 = 0$  ]