

DIFFERENTIATION

Answers

- 1 a $\frac{dy}{dx} = 2x \times (2-x)^3 + x^2 \times 3(2-x)^2 \times (-1)$
 $= x(2-x)^2(4-5x)$
 grad = -1
 $\therefore y-1 = -(x-1)$ [$y = 2-x$]
 b grad of normal = 1
 $\therefore y-1 = x-1$
 $y = x$
 when $x = 0, y = 0$ \therefore passes through origin
- 2 a $\frac{dy}{dx} = \frac{1 \times (2x+3) - x \times 2}{(2x+3)^2} = \frac{3}{(2x+3)^2}$
 grad = 3
 $\therefore y+1 = 3(x+1)$ [$y = 3x+2$]
 b at $(0, 0)$, grad = $\frac{1}{3}$
 \therefore grad of normal = -3
 $\therefore y = -3x$
 c $3x+2 = -3x$
 $x = -\frac{1}{3}$ $\therefore (-\frac{1}{3}, 1)$
- 3 a $P(-3, 0), Q(1, 0)$
- b $\frac{dy}{dx} = 1 \times (x-1)^3 + (x+3) \times 3(x-1)^2$
 $= (x-1)^2[(x-1) + 3(x+3)]$
 $= 4(x+2)(x-1)^2$
 SP: $4(x+2)(x-1)^2 = 0$
 $x = 1$ (at Q) or -2
 $\therefore R(-2, -27)$
- 4 a $\frac{dy}{dx} = 1 \times \sqrt{4x+1} + x \times \frac{1}{2}(4x+1)^{-\frac{1}{2}} \times 4$
 $= (4x+1)^{-\frac{1}{2}}[(4x+1) + 2x] = \frac{6x+1}{\sqrt{4x+1}}$
 b $\frac{6x+1}{\sqrt{4x+1}} - 5x\sqrt{4x+1} = 0$
 $6x+1 = 5x(4x+1)$
 $20x^2 - x - 1 = 0$
 $(5x+1)(4x-1) = 0$
 $x = -\frac{1}{5}, \frac{1}{4}$
- 5 a at $A, y = 0 \therefore x = 1$
 $\frac{dy}{dx} = \frac{2 \times (x^2+3) - 2(x-1) \times 2x}{(x^2+3)^2}$
 $= \frac{6+4x-2x^2}{(x^2+3)^2}$
 \therefore grad = $\frac{1}{2}$ \therefore grad of normal = -2
 $\therefore y-0 = -2(x-1)$
 $y = 2-2x$
 b SP: $\frac{6+4x-2x^2}{(x^2+3)^2} = 0$
 $2(1+x)(3-x) = 0$
 $x = -1, 3$
 $\therefore (-1, -1), (3, \frac{1}{3})$
- 6 a $f'(x) = \frac{3}{2}x^{\frac{1}{2}} \times (x-3)^3 + x^{\frac{3}{2}} \times 3(x-3)^2$
 $= \frac{3}{2}x^{\frac{1}{2}}(x-3)^2[(x-3) + 2x]$
 $= \frac{3}{2}x^{\frac{1}{2}}(3x-3)(x-3)^2$
 $= \frac{9}{2}x^{\frac{1}{2}}(x-1)(x-3)^2$ [$k = \frac{9}{2}$]
 b SP: $\frac{9}{2}x^{\frac{1}{2}}(x-1)(x-3)^2 = 0$
 $x > 0 \therefore x = 1, 3$
 $\therefore (1, -8), (3, 0)$
- 7 a $f'(x) = 1 \times \sqrt{2x+12} + x \times \frac{1}{2}(2x+12)^{-\frac{1}{2}} \times 2$
 $= (2x+12)^{-\frac{1}{2}}[(2x+12) + x]$
 $= \frac{3x+12}{\sqrt{2x+12}}$
 $f''(x) = \frac{3 \times \sqrt{2x+12} - (3x+12) \times \frac{1}{2}(2x+12)^{-\frac{1}{2}} \times 2}{2x+12}$
 $= \frac{3(2x+12) - (3x+12)}{(2x+12)^{\frac{3}{2}}} = \frac{3x+24}{(2x+12)^{\frac{3}{2}}}$
 $= \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$
- b SP: $\frac{3x+12}{\sqrt{2x+12}} = 0$
 $x = -4$
 $\therefore (-4, -8)$
 $f''(-4) = \frac{3}{2}$
 $f''(-4) > 0 \therefore$ minimum