| **DIFFERENTIATION** | *Answers* |

1 **a** $f(x) = x[x^3 + 3x^2(2) + 3x(2)^2 + 2^3]$
         $= x^4 + 6x^3 + 12x^2 + 8x$
     $f'(x) = 4x^3 + 18x^2 + 24x + 8$

b $f'(x) = 1 \times (x + 2)^3 + x \times 3(x + 2)^2$
      $= (x + 2)^2[(x + 2) + 3x]$
      $= 2(2x + 1)(x + 2)^2$

2 **a** $= 1 \times e^x + x \times e^x$
         $= e^x(1 + x)$

b $= 1 \times (x + 1)^5 + x \times 5(x + 1)^4$
      $= (x + 1)^4[(x + 1) + 5x]$
      $= (6x + 1)(x + 1)^4$

c $= 1 \times \ln x + x \times \dfrac{1}{x}$
     $= \ln x + 1$

d $= 2x \times (x - 1)^3 + x^2 \times 3(x - 1)^2$
      $= x(x - 1)^2[2(x - 1) + 3x]$
      $= x(5x - 2)(x - 1)^2$

e $= 3x^2 \times \ln 2x + x^3 \times \dfrac{1}{x}$
     $= x^2(3 \ln 2x + 1)$

f $= 2x \times e^{-x} + x^2 \times (-e^{-x})$
      $= xe^{-x}(2 - x)$

g $= 8x^3 \times (5 + x)^3 + 2x^4 \times 3(5 + x)^2$
     $= 2x^3(5 + x)^2[4(5 + x) + 3x]$
     $= 2x^3(20 + 7x)(5 + x)^2$

h $= 2x \times (x - 3)^4 + x^2 \times 4(x - 3)^3$
      $= 2x(x - 3)^3[(x - 3) + 2x]$
      $= 6x(x - 1)(x - 3)^3$

3 **a** $= 1 \times (2x - 1)^3 + x \times 3(2x - 1)^2 \times 2$
         $= (2x - 1)^2[(2x - 1) + 6x]$
         $= (8x - 1)(2x - 1)^2$

b $= 12x^3 \times e^{2x + 3} + 3x^4 \times e^{2x + 3} \times 2$
      $= 6x^3 e^{2x + 3}(2 + x)$

c $= 1 \times \sqrt{x - 1} + x \times \tfrac{1}{2}(x - 1)^{-\frac{1}{2}}$
     $= \tfrac{1}{2}(x - 1)^{-\frac{1}{2}}[2(x - 1) + x]$
     $= \dfrac{3x - 2}{2\sqrt{x - 1}}$

d $= 2x \times \ln(x + 6) + x^2 \times \dfrac{1}{x + 6}$
      $= 2x \ln(x + 6) + \dfrac{x^2}{x + 6}$

e $= 1 \times (1 - 5x)^4 + x \times 4(1 - 5x)^3 \times (-5)$
     $= (1 - 5x)^3[(1 - 5x) - 20x]$
     $= (1 - 25x)(1 - 5x)^3$

f $= 1 \times (x - 3)^3 + (x + 2) \times 3(x - 3)^2$
      $= (x - 3)^2[(x - 3) + 3(x + 2)]$
      $= (4x + 3)(x - 3)^2$

g $= \tfrac{4}{3}x^{\frac{1}{3}} \times e^{3x} + x^{\frac{4}{3}} \times 3e^{3x}$
     $= \tfrac{1}{3}x^{\frac{1}{3}} e^{3x}(4 + 9x)$

h $= 1 \times \ln(x^2 - 1) + (x + 1) \times \dfrac{1}{x^2 - 1} \times 2x$
      $= \ln(x^2 - 1) + \dfrac{2x(x + 1)}{(x + 1)(x - 1)}$
      $= \ln(x^2 - 1) + \dfrac{2x}{x - 1}$

i $= 2x \times \sqrt{3x + 1} + x^2 \times \tfrac{1}{2}(3x + 1)^{-\frac{1}{2}} \times 3$
     $= \tfrac{1}{2}x(3x + 1)^{-\frac{1}{2}}[4(3x + 1) + 3x]$
     $= \dfrac{x(15x + 4)}{2\sqrt{3x + 1}}$

4   **a** $f'(x) = 4 \times e^{3x} + 4x \times 3e^{3x}$
              $= 4e^{3x}(1 + 3x)$
       $f'(0) = 4 \times 1 \times 1 = 4$

                                                  b $f'(x) = 2 \times (x^2 + 2)^3 + 2x \times 3(x^2 + 2)^2 \times 2x$
                                                                      $= 2(x^2 + 2)^2[(x^2 + 2) + 6x^2]$
                                                                      $= 2(7x^2 + 2)(x^2 + 2)^2$
                                                  $f'(-1) = 2 \times 9 \times 9 = 162$

   c $f'(x) = 5 \times \ln 3x + (5x - 4) \times \dfrac{1}{x}$
                 $= 5 \ln 3x + 5 - \dfrac{4}{x}$
        $f'(\tfrac{1}{3}) = 0 + 5 - 12 = -7$

             d $f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}} \times (1 - 2x)^3 + x^{\frac{1}{2}} \times 3(1 - 2x)^2 \times (-2)$
                            $= \tfrac{1}{2}x^{-\frac{1}{2}}(1 - 2x)^2[(1 - 2x) - 12x]$
                            $= \tfrac{1}{2}x^{-\frac{1}{2}}(1 - 14x)(1 - 2x)^2$
                $f'(\tfrac{1}{4}) = \tfrac{1}{2} \times 2 \times (-\tfrac{5}{2}) \times \tfrac{1}{4} = -\tfrac{5}{8}$

5   **a** $\dfrac{dy}{dx} = 1 \times e^{2x} + x \times 2e^{2x}$
       SP: $e^{2x}(1 + 2x) = 0$
                $x = -\tfrac{1}{2}$
       $\therefore \; (-\tfrac{1}{2}, -\tfrac{1}{2}e^{-1})$

             b $\dfrac{dy}{dx} = 1 \times (x - 4)^3 + x \times 3(x - 4)^2$
                         $= (x - 4)^2[(x - 4) + 3x]$
             SP: $4(x - 1)(x - 4)^2 = 0$
                      $x = 1, 4$
             $\therefore \; (1, -27), (4, 0)$

   c $\dfrac{dy}{dx} = 2x \times (2x - 3)^4 + x^2 \times 4(2x - 3)^3 \times 2$
               $= 2x(2x - 3)^3[(2x - 3) + 4x]$
       SP: $6x(2x - 1)(2x - 3)^3 = 0$
                $x = 0, \tfrac{1}{2}, \tfrac{3}{2}$
       $\therefore \; (0, 0), (\tfrac{1}{2}, 4), (\tfrac{3}{2}, 0)$

             d $\dfrac{dy}{dx} = 1 \times \sqrt{x + 12} + x \times \tfrac{1}{2}(x + 12)^{-\frac{1}{2}}$
                       $= \tfrac{1}{2}(x + 12)^{-\frac{1}{2}}[2(x + 12) + x]$
             SP: $\dfrac{3(x + 8)}{2\sqrt{x + 12}} = 0$
                      $x = -8$
             $\therefore \; (-8, -16)$

   e $\dfrac{dy}{dx} = 2x \times e^{-4x} + x^2 \times (-4e^{-4x})$
       SP: $2xe^{-4x}(1 - 2x) = 0$
                $x = \tfrac{1}{2}$
       $\therefore \; (\tfrac{1}{2}, 2 + \tfrac{1}{4}e^{-2})$

             f $\dfrac{dy}{dx} = -3 \times (3 - x)^3 + (1 - 3x) \times 3(3 - x)^2 \times (-1)$
                       $= -3(3 - x)^2[(3 - x) + (1 - 3x)]$
             SP: $-12(1 - x)(3 - x)^2 = 0$
                      $x = 1, 3$
             $\therefore \; (1, -16), (3, 0)$

6   **a** $x = 1 \; \therefore \; y = 1$
       $\dfrac{dy}{dx} = 1 \times (x - 2)^4 + x \times 4(x - 2)^3$
              $= (x - 2)^3[(x - 2) + 4x]$
              $= (5x - 2)(x - 2)^3$
       grad $= -3$
       $\therefore \; y - 1 = -3(x - 1)$
         $[\, y = 4 - 3x \,]$

             b $x = 1 \; \therefore \; y = 3e$
                $\dfrac{dy}{dx} = 6x \times e^x + 3x^2 \times e^x$
                       $= 3xe^x(2 + x)$
             grad $= 9e$
             $\therefore \; y - 3e = 9e(x - 1)$
                $[\, y = 3e(3x - 2) \,]$

   c $x = \tfrac{1}{2} \; \therefore \; y = 0$
       $\dfrac{dy}{dx} = 4 \times \ln 2x + (4x - 1) \times \dfrac{1}{x}$
             $= 4 \ln 2x + 4 - \dfrac{1}{x}$
       grad $= 2$
       $\therefore \; y - 0 = 2(x - \tfrac{1}{2})$
         $[\, y = 2x - 1 \,]$

             d $x = -2 \; \therefore \; y = 8$
                $\dfrac{dy}{dx} = 2x \times \sqrt{x + 6} + x^2 \times \tfrac{1}{2}(x + 6)^{-\frac{1}{2}}$
                       $= \tfrac{1}{2}x(x + 6)^{-\frac{1}{2}}[4(x + 6) + x] = \dfrac{x(5x + 24)}{2\sqrt{x + 6}}$
             grad $= -7$
             $\therefore \; y - 8 = -7(x + 2)$
                $[\, y = -7x - 6 \,]$

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7 **a** $x = 1$ $\therefore$ $y = 1$

$\dfrac{dy}{dx} = 2x \times (2 - x)^3 + x^2 \times 3(2 - x)^2 \times (-1)$

$= x(2 - x)^2[2(2 - x) - 3x]$

$= x(4 - 5x)(2 - x)^2$

grad $= -1$

$\therefore$ grad of normal $= 1$

$\therefore$ $y - 1 = 1(x - 1)$

$x - y = 0$

b $x = 2$ $\therefore$ $y = 0$

$\dfrac{dy}{dx} = 1 \times \ln(3x - 5) + x \times \dfrac{1}{3x - 5} \times 3$

$= \ln(3x - 5) + \dfrac{3x}{3x - 5}$

grad $= 6$

$\therefore$ grad of normal $= -\frac{1}{6}$

$\therefore$ $y - 0 = -\frac{1}{6}(x - 2)$

$x + 6y - 2 = 0$

c $x = 0$ $\therefore$ $y = -1$

$\dfrac{dy}{dx} = 2x \times e^{3x} + (x^2 - 1) \times 3e^{3x}$

$= e^{3x}[2x + 3(x^2 - 1)]$

$= e^{3x}(3x^2 + 2x - 3)$

grad $= -3$

$\therefore$ grad of normal $= \frac{1}{3}$

$\therefore$ $y + 1 = \frac{1}{3}(x - 0)$

$x - 3y - 3 = 0$

d $x = 8$ $\therefore$ $y = 16$

$\dfrac{dy}{dx} = 1 \times \sqrt{x - 4} + x \times \frac{1}{2}(x - 4)^{-\frac{1}{2}}$

$= \frac{1}{2}(x - 4)^{-\frac{1}{2}}[2(x - 4) + x]$

$= \dfrac{3x - 8}{2\sqrt{x - 4}}$

grad $= 4$

$\therefore$ grad of normal $= -\frac{1}{4}$

$\therefore$ $y - 16 = -\frac{1}{4}(x - 8)$

$x + 4y - 72 = 0$

8 **a** $x = 1$ $\therefore$ $y = e$

$\dfrac{dy}{dx} = 1 \times e^{x^2} + x \times e^{x^2} \times 2x$

$= e^{x^2}(1 + 2x^2)$

grad $= 3e$

$\therefore$ $y - e = 3e(x - 1)$

$[\,y = e(3x - 2)\,]$

b $x = 0 \implies y = -2e$

$y = 0 \implies x = \frac{2}{3}$

$\therefore$ area $= \frac{1}{2} \times 2e \times \frac{2}{3} = \frac{2}{3}e$