# DIFFERENTIATION

**1** Given that $f(x) = x(x+2)^3$, find $f'(x)$

**a** by first expanding $f(x)$, **b** using the product rule.

**2** Differentiate each of the following with respect to $x$ and simplify your answers.

**a** $xe^x$ **b** $x(x+1)^5$ **c** $x\ln x$ **d** $x^2(x-1)^3$

**e** $x^3\ln 2x$ **f** $x^2e^{-x}$ **g** $2x^4(5+x)^3$ **h** $x^2(x-3)^4$

**3** Find $\frac{dy}{dx}$, simplifying your answer in each case.

**a** $y = x(2x-1)^3$ **b** $y = 3x^4e^{2x+3}$ **c** $y = x\sqrt{x-1}$

**d** $y = x^2\ln(x+6)$ **e** $y = x(1-5x)^4$ **f** $y = (x+2)(x-3)^3$

**g** $y = x^{\frac{4}{3}}e^{3x}$ **h** $y = (x+1)\ln(x^2-1)$ **i** $y = x^2\sqrt{3x+1}$

**4** Find the value of $f'(x)$ at the value of $x$ indicated in each case.

**a** $f(x) = 4xe^{3x}$, $x = 0$ **b** $f(x) = 2x(x^2+2)^3$, $x = -1$

**c** $f(x) = (5x-4)\ln 3x$, $x = \frac{1}{3}$ **d** $f(x) = x^{\frac{1}{2}}(1-2x)^3$, $x = \frac{1}{4}$

**5** Find the coordinates of any stationary points on each curve.

**a** $y = xe^{2x}$ **b** $y = x(x-4)^3$ **c** $y = x^2(2x-3)^4$

**d** $y = x\sqrt{x+12}$ **e** $y = 2 + x^2e^{-4x}$ **f** $y = (1-3x)(3-x)^3$

**6** Find an equation for the tangent to each curve at the point on the curve with the given $x$-coordinate.

**a** $y = x(x-2)^4$, $x = 1$ **b** $y = 3x^2e^x$, $x = 1$

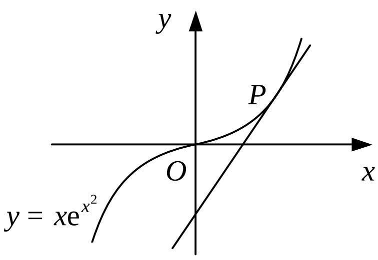
**c** $y = (4x-1)\ln 2x$, $x = \frac{1}{2}$ **d** $y = x^2\sqrt{x+6}$, $x = -2$

**7** Find an equation for the normal to each curve at the point on the curve with the given $x$-coordinate. Give your answers in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**a** $y = x^2(2-x)^3$, $x = 1$ **b** $y = x\ln(3x-5)$, $x = 2$

**c** $y = (x^2-1)e^{3x}$, $x = 0$ **d** $y = x\sqrt{x-4}$, $x = 8$

**8**




The diagram shows part of the curve with equation $y = xe^{x^2}$ and the tangent to the curve at the point $P$ with $x$-coordinate 1.

**a** Find an equation for the tangent to the curve at $P$.

**b** Show that the area of the triangle bounded by this tangent and the coordinate axes is $\frac{2}{3}e$.