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│  ⬡  DIFFERENTIATION                      Answers                  │
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1 **a** $= -\sin x$ **b** $= 5\cos x$

c $= -3\sin 3x$ **d** $= \frac{1}{4}\cos \frac{1}{4}x$

e $= \cos(x+1)$ **f** $= -3\sin(3x-2)$

g $= -4\cos(\frac{\pi}{3}-x)$ **h** $= -\frac{1}{2}\sin(\frac{1}{2}x+\frac{\pi}{6})$

i $= 2\sin x \cos x$ **j** $= 6\cos^2 x \times (-\sin x)$
$= -6\cos^2 x \sin x$

k $= 2\cos(x-1) \times [-\sin(x-1)]$ **l** $= 4\sin^3 2x \times 2\cos 2x$
$= -2\cos(x-1)\sin(x-1)$ $= 8\sin^3 2x \cos 2x$

2 **a** $= \dfrac{d}{dx}\left(\dfrac{\sin x}{\cos x}\right)$ **b** $= \dfrac{d}{dx}[(\cos x)^{-1}]$

$= \dfrac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$ $= -(\cos x)^{-2} \times (-\sin x)$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$ $= \dfrac{\sin x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$ $= \dfrac{1}{\cos x} \times \dfrac{\sin x}{\cos x}$

$= \sec^2 x$ $= \sec x \tan x$

c $= \dfrac{d}{dx}[(\sin x)^{-1}]$ **d** $= \dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right)$

$= -(\sin x)^{-2} \times \cos x$ $= \dfrac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$

$= -\dfrac{\cos x}{\sin^2 x}$ $= -\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}$

$= -\dfrac{1}{\sin x} \times \dfrac{\cos x}{\sin x}$ $= -\dfrac{1}{\sin^2 x}$

$= -\operatorname{cosec} x \cot x$ $= -\operatorname{cosec}^2 x$

3 **a** $= -2\operatorname{cosec}^2 2t$ **b** $= \sec(t+2)\tan(t+2)$

c $= 4\sec^2(4t-3)$ **d** $= -3\operatorname{cosec} 3t \cot 3t$

e $= 2\tan t \times \sec^2 t$ **f** $= -3\operatorname{cosec}(t+\frac{\pi}{6})\cot(t+\frac{\pi}{6})$
$= 2\tan t \sec^2 t$

g $= 3\cot^2 t \times (-\operatorname{cosec}^2 t)$ **h** $= 2\sec \frac{1}{2}t \tan \frac{1}{2}t$
$= -3\cot^2 t \operatorname{cosec}^2 t$

i $= -2\operatorname{cosec}^2(2t-3)$ **j** $= 2\sec 2t \times 2\sec 2t \tan 2t$
$= 4\sec^2 2t \tan 2t$

k $= -2\sec^2(\pi-4t)$ **l** $= 2\operatorname{cosec}(3t+1) \times -3\operatorname{cosec}(3t+1)\cot(3t+1)$
$= -6\operatorname{cosec}^2(3t+1)\cot(3t+1)$

4   **a** $= \dfrac{1}{\sin x} \times \cos x$ **b** $= 6e^{\tan x} \times \sec^2 x$

        $= \cot x$ $= 6e^{\tan x} \sec^2 x$

    c $= \tfrac{1}{2}(\cos 2x)^{-\frac{1}{2}} \times (-2\sin 2x)$ **d** $= e^{\sin 3x} \times 3\cos 3x$

        $= -\dfrac{\sin 2x}{\sqrt{\cos 2x}}$ $= 3e^{\sin 3x} \cos 3x$

    e $= -2\operatorname{cosec}^2 x^2 \times 2x$ **f** $= \tfrac{1}{2}(\sec x)^{-\frac{1}{2}} \times \sec x \tan x$

        $= -4x \operatorname{cosec}^2 x^2$ $= \tfrac{1}{2}\sqrt{\sec x}\,\tan x$

    g $= 3e^{-\operatorname{cosec} 2x} \times 2\operatorname{cosec} 2x \cot 2x$ **h** $= \dfrac{1}{\tan 4x} \times 4\sec^2 4x$

        $= 6\operatorname{cosec} 2x \cot 2x\, e^{-\operatorname{cosec} 2x}$ $= \dfrac{\cos 4x}{\sin 4x} \times \dfrac{4}{\cos^2 4x}$

                                                  $= 4\sec 4x \operatorname{cosec} 4x$

5   **a** $\dfrac{dy}{dx} = 1 + 2\cos x$ **b** $\dfrac{dy}{dx} = 2\sec x \tan x - \sec^2 x$ **c** $\dfrac{dy}{dx} = \cos x - 2\sin 2x$

       SP: $1 + 2\cos x = 0$ $= \sec x(2\tan x - \sec x)$ SP: $\cos x - 2\sin 2x = 0$

               $\cos x = -\tfrac{1}{2}$ SP: $\sec x(2\tan x - \sec x) = 0$ $\cos x - 4\sin x\cos x = 0$

                $x = \pi - \tfrac{\pi}{3},\ \pi + \tfrac{\pi}{3}$ $2\tan x - \sec x = 0$ $\cos x(1 - 4\sin x) = 0$

                $x = \tfrac{2\pi}{3},\ \tfrac{4\pi}{3}$ $\dfrac{2\sin x}{\cos x} = \dfrac{1}{\cos x}$ $\cos x = 0$ or $\sin x = \tfrac{1}{4}$

       $\therefore\ (\tfrac{2\pi}{3},\ \tfrac{2\pi}{3} + \sqrt{3}\,)$, $\sin x = \tfrac{1}{2}$ $x = \tfrac{\pi}{2},\ \tfrac{3\pi}{2}$ or $0.253,\ \pi - 0.25268$

              $(\tfrac{4\pi}{3},\ \tfrac{4\pi}{3} - \sqrt{3}\,)$ $x = \tfrac{\pi}{6},\ \pi - \tfrac{\pi}{6}$ $x = 0.253$ (3sf), $\tfrac{\pi}{2}$, 2.89 (3sf), $\tfrac{3\pi}{2}$

                                        $x = \tfrac{\pi}{6},\ \tfrac{5\pi}{6}$ $\therefore\ (0.253,\ \tfrac{9}{8}),\ (\tfrac{\pi}{2},\ 0),$

                          $\therefore\ (\tfrac{\pi}{6},\ \sqrt{3}\,),\ (\tfrac{5\pi}{6},\ -\sqrt{3}\,)$ $(2.89,\ \tfrac{9}{8}),\ (\tfrac{3\pi}{2},\ -2)$

6   **a** $x = 0\ \therefore\ y = 1$ **b** $x = \tfrac{\pi}{3}\ \therefore\ y = \tfrac{1}{2}$

        $\dfrac{dy}{dx} = 2\cos 2x$ $\dfrac{dy}{dx} = -\sin x$

        $\text{grad} = 2$ $\text{grad} = -\dfrac{\sqrt{3}}{2}$

        $\therefore\ y - 1 = 2(x - 0)$ $\therefore\ y - \tfrac{1}{2} = -\dfrac{\sqrt{3}}{2}(x - \tfrac{\pi}{3})$

          $[\,y = 2x + 1\,]$ $[\,3\sqrt{3}\,x + 6y - 3 - \sqrt{3}\,\pi = 0\,]$

    c $x = \tfrac{\pi}{4}\ \therefore\ y = -1$ **d** $x = \tfrac{\pi}{6}\ \therefore\ y = 1$

        $\dfrac{dy}{dx} = 3\sec^2 3x$ $\dfrac{dy}{dx} = -\operatorname{cosec} x \cot x - 2\cos x$

        $\text{grad} = 6$ $\text{grad} = -3\sqrt{3}$

        $\therefore\ y + 1 = 6(x - \tfrac{\pi}{4})$ $\therefore\ y - 1 = -3\sqrt{3}\,(x - \tfrac{\pi}{6})$

         $[\,12x - 2y - 2 - 3\pi = 0\,]$ $[\,6\sqrt{3}\,x + 2y - 2 - \sqrt{3}\,\pi = 0\,]$

7   **a** $= 1 \times \sin x + x \times \cos x$

          $= \sin x + x \cos x$

   b $= \dfrac{-2\sin 2x \times x - \cos 2x \times 1}{x^2}$

          $= -\dfrac{2x\sin 2x + \cos 2x}{x^2}$

   c $= e^x \times \cos x + e^x \times (-\sin x)$
          $= e^x(\cos x - \sin x)$

   d $= \cos x \times \cos x + \sin x \times (-\sin x)$
          $= \cos^2 x - \sin^2 x = \cos 2x$

   e $= 2x \times \operatorname{cosec} x + x^2 \times (-\operatorname{cosec} x \cot x)$
          $= x \operatorname{cosec} x(2 - x \cot x)$

   f $= \sec x \tan x \times \tan x + \sec x \times \sec^2 x$
          $= \sec x(\tan^2 x + \sec^2 x)$

   g $= \dfrac{1 \times \tan x - x \times \sec^2 x}{\tan^2 x}$

          $= \cot x - x \operatorname{cosec}^2 x$

   h $= \dfrac{2\cos 2x \times e^{3x} - \sin 2x \times 3e^{3x}}{(e^{3x})^2}$

          $= \dfrac{2\cos 2x - 3\sin 2x}{e^{3x}}$

   i $= 2\cos x(-\sin x) \times \cot x + \cos^2 x \times (-\operatorname{cosec}^2 x)$

          $= -2\cos^2 x - \cot^2 x$

   j $= \dfrac{2\sec 2x \tan 2x \times x^2 - \sec 2x \times 2x}{x^4}$

          $= \dfrac{2\sec 2x(x \tan 2x - 1)}{x^3}$

   k $= 1 \times \tan^2 4x + x \times 2 \tan 4x \times 4 \sec^2 4x$

          $= \tan 4x(\tan 4x + 8x \sec^2 4x)$

   l $= \dfrac{\cos x \times \cos 2x - \sin x \times (-2\sin 2x)}{\cos^2 2x}$

          $= \dfrac{\cos x(1 - 2\sin^2 x) + 4\sin^2 x \cos x}{\cos^2 2x} = \dfrac{\cos x(1 + 2\sin^2 x)}{\cos^2 2x}$

8   **a** $f'(x) = 3\cos 3x \times \cos 5x + \sin 3x \times (-5\sin 5x)$
              $= 3 \cos 3x \cos 5x - 5 \sin 3x \sin 5x$

          $f'(\frac{\pi}{4}) = 3(-\frac{1}{\sqrt 2})(-\frac{1}{\sqrt 2}) - 5 \times \frac{1}{\sqrt 2}(-\frac{1}{\sqrt 2})$

              $= \frac{3}{2} + \frac{5}{2} = 4$

   b $f'(x) = 2 \sec^2 2x \times \sin x + \tan 2x \times \cos x$
            $= 2 \sec^2 2x \sin x + \tan 2x \cos x$

         $f'(\frac{\pi}{3}) = 2 \times 4 \times \frac{\sqrt 3}{2} + (-\sqrt 3) \times \frac{1}{2}$

            $= 4\sqrt 3 - \frac{1}{2}\sqrt 3 = \frac{7}{2}\sqrt 3$

   c $f'(x) = \dfrac{\frac{1}{2\cos x} \times (-2\sin x) \times \sin x - \ln(2\cos x) \times \cos x}{\sin^2 x}$

          $= -\sec x - \dfrac{\cos x \ln(2\cos x)}{\sin^2 x}$

         $f'(\frac{\pi}{3}) = -2 - 0 = -2$

   d $f'(x) =$

     $2\sin x \cos x \times \cos^3 x + \sin^2 x \times 3\cos^2 x \times (-\sin x)$

          $= \sin x \cos^2 x(2 \cos^2 x - 3 \sin^2 x)$

     $f'(\frac{\pi}{6}) = \frac{1}{2} \times \frac{3}{4}(2 \times \frac{3}{4} - 3 \times \frac{1}{4}) = \frac{9}{32}$

9 $x = 0 \;\; \therefore y = 3$

    $\dfrac{dy}{dx} = 1 \times \cos 2x + x \times (-2 \sin 2x)$

       $= \cos 2x - 2x \sin 2x$

    grad $= 1$

    $\therefore$ grad of normal $= -1$

    $\therefore y - 3 = -(x - 0)$

     $[\, y = 3 - x \,]$

10   **a** $= \dfrac{\cos x \times (1 - \sin x) - (2 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$

          $= \dfrac{3\cos x}{(1 - \sin x)^2}$

   b   SP: $\dfrac{3\cos x}{(1 - \sin x)^2} = 0$

           $\cos x = 0$

       $x \neq \frac{\pi}{2} \;\; \therefore x = \frac{3\pi}{2}$

         $\therefore (\frac{3\pi}{2}, \frac{1}{2})$

   c $x = \frac{\pi}{6} \;\; \therefore y = 5$

       grad $= 6\sqrt 3$

       $y - 5 = 6\sqrt 3 (x - \frac{\pi}{6})$
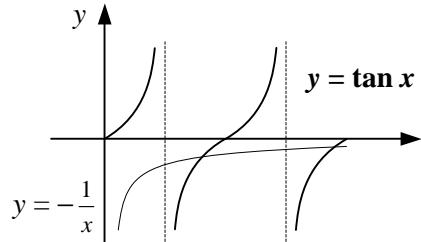
       $y = 6\sqrt 3 x + 5 - \sqrt 3 \pi$

11 **a** $\dfrac{dy}{dx} = -e^{-x} \times \sin x + e^{-x} \times \cos x$

$\qquad = e^{-x}(\cos x - \sin x)$

$\qquad \dfrac{d^2y}{dx^2} = -e^{-x} \times (\cos x - \sin x)$

$\qquad\qquad\quad + e^{-x} \times (-\sin x - \cos x)$

$\qquad\qquad = -2e^{-x}\cos x$

b SP: $e^{-x}(\cos x - \sin x) = 0$

$\qquad\quad \cos x - \sin x = 0$

$\qquad\quad \tan x = 1$

$\qquad\quad x = \frac{\pi}{4}, \ \frac{\pi}{4} - \pi = -\frac{3\pi}{4}, \ \frac{\pi}{4}$

$\qquad x = -\frac{3\pi}{4}: \ \dfrac{d^2y}{dx^2} = \sqrt{2}\,e^{\frac{3\pi}{4}} \ (>0)$

$\qquad x = \frac{\pi}{4}: \ \dfrac{d^2y}{dx^2} = -\sqrt{2}\,e^{-\frac{\pi}{4}} \ (<0)$

$\qquad \therefore \ (-\frac{3\pi}{4}, \ -\frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}})$, minimum

$\qquad\quad (\frac{\pi}{4}, \ \frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}})$, maximum

12 **a** $\dfrac{dy}{dx} = 1 \times \sec x + x \times \sec x \tan x$

$\qquad = \sec x(1 + x\tan x)$

$\qquad$ SP: $\sec x(1 + x\tan x) = 0$

$\qquad$ no real values of x for which $\sec x = 0$

$\qquad \therefore \qquad 1 + x\tan x = 0$

b $1 + x\tan x = 0$

$\qquad \Rightarrow \quad x\tan x = -1$

$\qquad\qquad\quad \tan x = -\dfrac{1}{x}$

$\qquad \therefore$ x-coord of SP where curves

$\qquad\qquad y = \tan x$ and $y = -\dfrac{1}{x}$ intersect

$\qquad$ intersect at 2 points $\therefore$ 2 SP in interval

13 **a** $f'(x) = -\sin x \times \sin 2x + \cos x \times 2\cos 2x$

$\qquad = 2\cos x(1 - 2\sin^2 x) - 2\sin^2 x \cos x$

$\qquad = 2\cos x(1 - 3\sin^2 x)$

b SP: $2\cos x(1 - 3\sin^2 x) = 0$

$\qquad\quad \cos x = 0$ or $\sin x = \pm\dfrac{1}{\sqrt{3}}$

$\qquad x = \frac{\pi}{2}, \ \frac{3\pi}{2}$ or $0.615, \ \pi - 0.61548,$

$\qquad\qquad\qquad \pi + 0.61548, \ 2\pi - 0.61548$

$\qquad x = 0.615, \ \frac{\pi}{2}, \ 2.53, \ 3.76, \ \frac{3\pi}{2}, \ 5.67$

c using graph, max. f(x) when $\sin x = \dfrac{1}{\sqrt{3}}$

$\qquad \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$\qquad f(x) = \cos x \times 2\sin x \cos x = 2\sin x \cos^2 x$

$\qquad \therefore$ max. f(x) $= 2 \times \dfrac{1}{\sqrt{3}} \times \dfrac{2}{3}$

$\qquad\qquad = \dfrac{4}{3\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4}{9}\sqrt{3}$

d period of $\cos x = 2\pi$, period of $\sin 2x = \pi$

$\qquad \therefore$ period of f(x) $= 2\pi$

$\qquad \therefore$ values of f(x) in this interval are repeated

14 **a** $x = 0 \ \therefore \ y = -2$

$\qquad \dfrac{dy}{dx} = -\text{cosec}\,(x - \frac{\pi}{6})\cot(x - \frac{\pi}{6})$

$\qquad \text{grad} = -2\sqrt{3}$

$\qquad \therefore$ grad of normal $= \dfrac{1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

$\qquad \therefore \ y + 2 = \frac{1}{6}\sqrt{3}\,(x - 0)$

$\qquad\quad [\ y = \frac{1}{6}\sqrt{3}\,x - 2\ ]$

b $x = \frac{\pi}{3} \ \therefore \ y = 2$

$\qquad \text{grad} = -2\sqrt{3}$

$\qquad \therefore \ y - 2 = -2\sqrt{3}\,(x - \frac{\pi}{3})$

$\qquad\quad [\ 6\sqrt{3}\,x + 3y - 6 - 2\sqrt{3}\,\pi = 0\ ]$

c $\frac{1}{6}\sqrt{3}\,x - 2 = 2 - 2\sqrt{3}\,(x - \frac{\pi}{3})$

$\qquad x - 4\sqrt{3} = 4\sqrt{3} - 12x + 4\pi$

$\qquad 13x = 8\sqrt{3} + 4\pi$

$\qquad x = \dfrac{8\sqrt{3} + 4\pi}{13}$