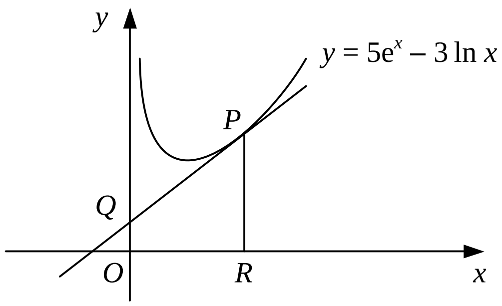
DIFFERENTIATION

1 **a** Find an equation for the normal to the curve $y = \frac{2}{5}x + \frac{1}{10}e^x$ at the point on the curve where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point where this normal crosses the x-axis.

2



The diagram shows the curve with equation $y = 5e^x - 3 \ln x$ and the tangent to the curve at the point P with x-coordinate 1.

a Show that the tangent at P has equation $y = (5e - 3)x + 3$.

The tangent at P meets the y-axis at Q.
The line through P parallel to the y-axis meets the x-axis at R.

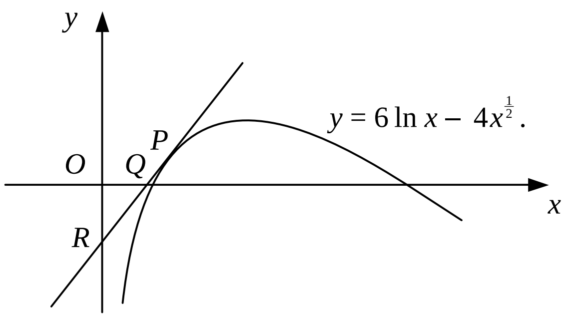
b Find the area of trapezium $ORPQ$, giving your answer in terms of e.

3 A curve has equation $y = 3x - \frac{1}{2}e^x$.

a Find the coordinates of the stationary point on the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

4



The diagram shows the curve $y = 6 \ln x - 4x^{\frac{1}{2}}$. The x-coordinate of the point P on the curve is 4.

The tangent to the curve at P meets the x-axis at Q and the y-axis at R.

a Find an equation for the tangent to the curve at P.

b Hence, show that the area of triangle OQR is $(10 - 12 \ln 2)^2$.

5 The curve with equation $y = 2x - 2 - \ln x$ passes through the point A (1, 0). The tangent to the curve at A crosses the y-axis at B and the normal to the curve at A crosses the y-axis at C.

a Find an equation for the tangent to the curve at A.

b Show that the mid-point of BC is the origin.

The curve has a minimum point at D.

c Show that the y-coordinate of D is $\ln 2 - 1$.

6 **a** Sketch the curve with equation $y = e^x + k$, where k is a positive constant.

Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

b Find an equation for the tangent to the curve at the point on the curve where $x = 2$.

Given that the tangent passes through the x-axis at the point $(-1, 0)$,

c show that $k = 2e^2$.

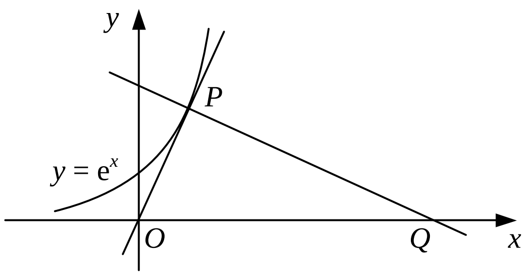
7 A curve has equation $y = 3x^2 - 2 \ln x$, $x > 0$.

The gradient of the curve at the point P on the curve is -1.

a Find the x-coordinate of P.

b Find an equation for the tangent to the curve at the point on the curve where $x = 1$.

8



The diagram shows the curve with equation $y = e^x$ which passes through the point P (p, e^p). Given that the tangent to the curve at P passes through the origin and that the normal to the curve at P meets the x-axis at Q,

a show that $p = 1$,

b show that the area of triangle OPQ, where O is the origin, is $\frac{1}{2}e(1 + e^2)$.

9 The curve with equation $y = 4 - e^x$ meets the y-axis at the point P and the x-axis at the point Q.

a Find an equation for the normal to the curve at P.

b Find an equation for the tangent to the curve at Q.

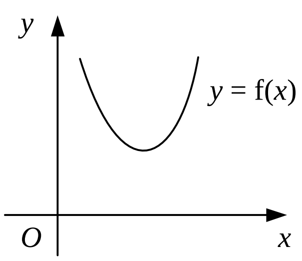
The normal to the curve at P meets the tangent to the curve at Q at the point R.

The x-coordinate of R is $a \ln 2 + b$, where a and b are rational constants.

c Show that $a = \frac{8}{5}$.

d Find the value of b.

10



The diagram shows a sketch of the curve $y = f(x)$ where

$$f : x \to 9x^4 - 16 \ln x, \quad x > 0.$$

Given that the set of values of x for which $f(x)$ is a decreasing function of x is $0 < x \leq k$, find the exact value of k.