DIFFERENTIATION — *Answers*

1 **a** $x = 0 \;\therefore\; y = \frac{1}{10}$

$\frac{dy}{dx} = \frac{2}{5} + \frac{1}{10}e^x$, grad $= \frac{1}{2}$

$\therefore$ grad of normal $= -2$

$\therefore\; y = -2x + \frac{1}{10}$

$20x + 10y - 1 = 0$

b $y = 0 \;\therefore\; x = \frac{1}{20}$

$(\frac{1}{20}, 0)$

2 **a** $x = 1 \;\therefore\; y = 5e$

$\frac{dy}{dx} = 5e^x - \frac{3}{x}$, grad $= 5e - 3$

$\therefore\; y - 5e = (5e - 3)(x - 1)$

$y = (5e - 3)x + 3$

b at Q, $x = 0 \;\therefore\; y = 3$

R is $(1, 0)$

area $= \frac{1}{2} \times (3 + 5e) \times 1$

$= \frac{1}{2}(5e + 3)$

3 **a** $\frac{dy}{dx} = 3 - \frac{1}{2}e^x$

SP: $3 - \frac{1}{2}e^x = 0$

$x = \ln 6$

$\therefore\; (\ln 6, 3\ln 6 - 3)$

b $\frac{d^2y}{dx^2} = -\frac{1}{2}e^x$

$x = \ln 6$: $\frac{d^2y}{dx^2} = -3$

$\therefore$ max

4 **a** at P, $x = 4 \;\therefore\; y = 6\ln 4 - 8$

$\frac{dy}{dx} = \frac{6}{x} - 2x^{-\frac{1}{2}}$, grad $= \frac{1}{2}$

$\therefore\; y - (6\ln 4 - 8) = \frac{1}{2}(x - 4)$

$[\, y = \frac{1}{2}x - 10 + 12\ln 2 \,]$

b at Q, $y = 0 \;\therefore\; x = 20 - 24\ln 2$

at R, $x = 0 \;\therefore\; y = 12\ln 2 - 10$

area $= \frac{1}{2} \times (20 - 24\ln 2) \times (10 - 12\ln 2)$

$= (10 - 12\ln 2)^2$

5 **a** $\frac{dy}{dx} = 2 - \frac{1}{x}$, grad $= 1$

$\therefore\; y = x - 1$

b grad of normal $= -1$

$\therefore\; y = -(x - 1)$ $\quad [\, y = 1 - x \,]$

at B, $x = 0 \;\therefore\; y = -1$

at C, $x = 0 \;\therefore\; y = 1$

mid-point of $(0, -1)$ and $(0, 1)$

$= (0, \frac{-1+1}{2}) = (0, 0)$

$\therefore$ mid-point of BC is the origin

c SP: $2 - \frac{1}{x} = 0$

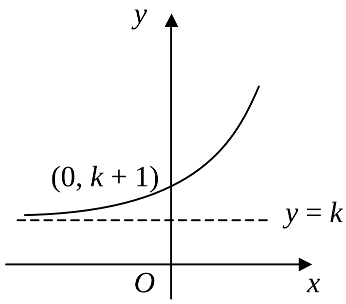
$x = \frac{1}{2}$

$\therefore\; y = 1 - 2 - \ln \frac{1}{2}$

$= -1 - \ln 2^{-1}$

$= \ln 2 - 1$

6 **a**



b $x = 2 \;\therefore\; y = e^2 + k$

$\frac{dy}{dx} = e^x$, grad $= e^2$

$\therefore\; y - (e^2 + k) = e^2(x - 2)$

$[\, y = e^2x - e^2 + k \,]$

c $(-1, 0) \;\therefore\; 0 = -e^2 - e^2 + k$

$k = 2e^2$

7 **a** $\frac{dy}{dx} = 6x - \frac{2}{x}$

at P, $6x - \frac{2}{x} = -1$

$6x^2 + x - 2 = 0$

$(3x + 2)(2x - 1) = 0$

$x > 0 \therefore x = \frac{1}{2}$

b $x = 1 \therefore y = 3$, grad = 4

$\therefore y - 3 = 4(x - 1)$

[$y = 4x - 1$]

8 **a** $\frac{dy}{dx} = e^x$, grad at $P = e^p$

tangent: $y - e^p = e^p(x - p)$

(0, 0) $\therefore$ $0 - e^p = e^p(0 - p)$

$e^p(p - 1) = 0$

$e^p \neq 0 \therefore p = 1$

b P (1, e), grad at P = e

$\therefore$ grad of normal $= -\frac{1}{e}$

$\therefore y - e = -\frac{1}{e}(x - 1)$

at Q, $y = 0 \therefore x = e^2 + 1$

$\therefore$ area $= \frac{1}{2} \times (e^2 + 1) \times e = \frac{1}{2}e(1 + e^2)$

9 **a** at P, $x = 0 \therefore y = 3$

$\frac{dy}{dx} = -e^x$, grad = -1

$\therefore$ grad of normal = 1

$\therefore y = x + 3$

b at Q, $y = 0 \therefore x = \ln 4$

grad at $Q = -4$

$\therefore y = -4(x - \ln 4)$ [$y = 8 \ln 2 - 4x$]

c at R $x + 3 = -4(x - \ln 4)$

$5x = 4 \ln 4 - 3 = 8 \ln 2 - 3$

$x = \frac{1}{5}(8 \ln 2 - 3)$

$\therefore a = \frac{8}{5}$

d $b = -\frac{3}{5}$

10 $f'(x) = 36x^3 - \frac{16}{x}$

SP: $36x^3 - \frac{16}{x} = 0$

$x^4 = \frac{4}{9}$

$x^2 = -\frac{2}{3}$ [no solutions] or $\frac{2}{3}$

$x > 0 \therefore x = \sqrt{\frac{2}{3}}$

$\therefore$ decreasing for $0 < x \leq \sqrt{\frac{2}{3}}$

$k = \sqrt{\frac{2}{3}}$ or $\frac{1}{3}\sqrt{6}$