DIFFERENTIATION — *Answers*

1 **a** e^x **b** $3e^x$ **c** $\frac{1}{x}$ **d** $\frac{1}{2x}$

2 **a** $-2e^t$ **b** $6t + \frac{1}{t}$ **c** $e^t + 5t^4$ **d** $\frac{3}{2}t^{\frac{1}{2}} + 2e^t$

e $\frac{2}{t} + \frac{1}{2}t^{-\frac{1}{2}}$ or $\frac{2}{t} + \frac{1}{2\sqrt{t}}$ **f** $2.5e^t - \frac{7}{2t}$ **g** $-t^{-2} + \frac{8}{t}$ or $\frac{8}{t} - \frac{1}{t^2}$ **h** $14t - 2 + 4e^t$

3 **a** $\frac{dy}{dx} = 12x^2 + e^x$, $\frac{d^2y}{dx^2} = 24x + e^x$

b $\frac{dy}{dx} = 7e^x - 10x + 3$, $\frac{d^2y}{dx^2} = 7e^x - 10$

c $\frac{dy}{dx} = \frac{1}{x} + \frac{5}{2}x^{\frac{3}{2}}$, $\frac{d^2y}{dx^2} = -x^{-2} + \frac{15}{4}x^{\frac{1}{2}}$

d $\frac{dy}{dx} = 5e^x + \frac{6}{x}$, $\frac{d^2y}{dx^2} = 5e^x - 6x^{-2}$

e $\frac{dy}{dx} = -3x^{-2} + \frac{3}{x}$, $\frac{d^2y}{dx^2} = 6x^{-3} - 3x^{-2}$

f $\frac{dy}{dx} = 2x^{-\frac{1}{2}} + \frac{1}{4x}$, $\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} - \frac{1}{4}x^{-2}$

4 **a** $f'(x) = 3 + e^x$

$f'(0) = 3 + 1 = 4$

b $f'(x) = \frac{1}{x} - 2x$

$f'(4) = \frac{1}{4} - 8 = -7\frac{3}{4}$

c $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{x}$

$f'(9) = \frac{1}{6} + \frac{2}{9} = \frac{7}{18}$

d $f'(x) = 5e^x - 2x^{-3}$

$f'(-\frac{1}{2}) = 5e^{-\frac{1}{2}} + 16$ [19.0 (3sf)]

5 **a** $\frac{dy}{dx} = \frac{5}{x} - 8 = 0$

$5 = 8x$

$x = \frac{5}{8}$

b $\frac{dy}{dx} = 2.4e^x - 3.6 = 0$

$e^x = 1.5$

$x = \ln 1.5$

[0.405 (3sf)]

c $\frac{dy}{dx} = 6x - 14 + \frac{4}{x} = 0$

$3x^2 - 7x + 2 = 0$

$(3x - 1)(x - 2) = 0$

$x = \frac{1}{3}, 2$

6 **a** $f'(x) = 2e^x - 3 = 7$

$e^x = 5$

$x = \ln 5$ [1.61 (3sf)]

b $f'(x) = 15 + \frac{1}{x} = 23$

$\frac{1}{x} = 8$

$x = \frac{1}{8}$

c $f'(x) = \frac{1}{4}x - 2 + \frac{1}{x} = -1$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$x = 2$

d $f'(x) = \frac{30}{x} - 2x = 4$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$\ln x$ only real for $x > 0$ $\therefore x = 3$

7 **a** $\frac{dy}{dx} = e^x - 2$

SP: $e^x - 2 = 0$

$x = \ln 2$

$\frac{d^2y}{dx^2} = e^x$

$x = \ln 2$: $\frac{d^2y}{dx^2} = 2$

$\therefore$ $(\ln 2, 2 - 2\ln 2)$, min

b $\frac{dy}{dx} = \frac{1}{x} - 10$

SP: $\frac{1}{x} - 10 = 0$

$x = \frac{1}{10}$

$\frac{d^2y}{dx^2} = -x^{-2}$

$x = \frac{1}{10}$: $\frac{d^2y}{dx^2} = -100$

$\therefore$ $(\frac{1}{10}, -1 - \ln 10)$, max

c $\frac{dy}{dx} = \frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}}$

SP: $\frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}} = 0$

$4 - x^{\frac{1}{2}} = 0$

$x^{\frac{1}{2}} = 4,\ x = 16$

$\frac{d^2y}{dx^2} = -2x^{-2} + \frac{1}{4}x^{-\frac{3}{2}}$

$x = 16$: $\frac{d^2y}{dx^2} = -\frac{1}{256}$

$\therefore$ $(16, 8\ln 2 - 4)$, max

d $\frac{dy}{dx} = 4 - 5e^x$

SP: $4 - 5e^x = 0$

$x = \ln \frac{4}{5}$

$\frac{d^2y}{dx^2} = -5e^x$

$x = \ln \frac{4}{5}$: $\frac{d^2y}{dx^2} = -4$

$\therefore$ $(\ln \frac{4}{5}, 4\ln \frac{4}{5} - 4)$, max

e $\frac{dy}{dx} = 2 - \frac{4}{x}$

SP: $2 - \frac{4}{x} = 0$

$x = 2$

$\frac{d^2y}{dx^2} = 4x^{-2}$

$x = 2$: $\frac{d^2y}{dx^2} = 1$

$\therefore$ $(2, 11 - 4\ln 2)$, min

f $\frac{dy}{dx} = 2x - 26 + \frac{72}{x}$

SP: $2x - 26 + \frac{72}{x} = 0$

$x^2 - 13x + 36 = 0$

$(x - 4)(x - 9) = 0$

$x = 4, 9$

$\frac{d^2y}{dx^2} = 2 - 72x^{-2}$

$x = 4$: $\frac{d^2y}{dx^2} = -\frac{5}{2}$

$x = 9$: $\frac{d^2y}{dx^2} = \frac{10}{9}$

$\therefore$ $(4, 144\ln 2 - 88)$, max

$(9, 144\ln 3 - 153)$, min

8 $\frac{dy}{dx} = 1 + ke^x$

$\frac{d^2y}{dx^2} = ke^x$

$\therefore$ $(1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1 - x)ke^x + x(1 + ke^x) - (x + ke^x)$

$= ke^x - kxe^x + x + kxe^x - x - ke^x = 0$

9 **a** $x = 2 \ \therefore \ y = e^2$

$\frac{dy}{dx} = e^x$, grad $= e^2$

$\therefore \ y - e^2 = e^2(x - 2)$

$[\ y = e^2(x - 1)\ ]$

b $x = 3 \ \therefore \ y = \ln 3$

$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{3}$

$\therefore \ y - \ln 3 = \frac{1}{3}(x - 3)$

$[\ y = \frac{1}{3}x + \ln 3 - 1\ ]$

c $x = 0 \ \therefore \ y = -2$

$\frac{dy}{dx} = 0.8 - 2e^x$, grad $= -1.2$

$\therefore \ y = -1.2x - 2$

d $x = 1 \ \therefore \ y = 4$

$\frac{dy}{dx} = \frac{5}{x} - 4x^{-2}$, grad $= 1$

$\therefore \ y - 4 = x - 1$

$[\ y = x + 3\ ]$

e $x = 1 \ \therefore \ y = 1 - 3e$

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 3e^x$, grad $= \frac{1}{3} - 3e$

$\therefore \ y - (1 - 3e) = (\frac{1}{3} - 3e)(x - 1)$

$[\ y = (\frac{1}{3} - 3e)x + \frac{2}{3}\ ]$

f $x = 9 \ \therefore \ y = \ln 9 - 3$

$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2}x^{-\frac{1}{2}}$, grad $= -\frac{1}{18}$

$\therefore \ y - (\ln 9 - 3) = -\frac{1}{18}(x - 9)$

$[\ y = \ln 9 - \frac{5}{2} - \frac{1}{18}x\ ]$

10 **a** $x = e \ \therefore \ y = 1$

$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{e}$

$\therefore$ grad of normal $= -e$

$\therefore \ y - 1 = -e(x - e)$

$[\ y = e^2 + 1 - ex\ ]$

b $x = 0 \ \therefore \ y = 7$

$\frac{dy}{dx} = 3e^x$, grad $= 3$

$\therefore$ grad of normal $= -\frac{1}{3}$

$\therefore \ y = 7 - \frac{1}{3}x$

c $x = 3 \ \therefore \ y = 10 + \ln 3$

$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{3}$

$\therefore$ grad of normal $= -3$

$\therefore \ y - (10 + \ln 3) = -3(x - 3)$

$[\ y = 19 + \ln 3 - 3x\ ]$

d $x = 1 \ \therefore \ y = -2$

$\frac{dy}{dx} = \frac{3}{x} - 2$, grad $= 1$

$\therefore$ grad of normal $= -1$

$\therefore \ y + 2 = -(x - 1)$

$[\ y = -x - 1\ ]$

e $x = 1 \ \therefore \ y = 1$

$\frac{dy}{dx} = 2x + \frac{8}{x}$, grad $= 10$

$\therefore$ grad of normal $= -\frac{1}{10}$

$\therefore \ y - 1 = -\frac{1}{10}(x - 1)$

$[\ y = \frac{1}{10}(11 - x)\ ]$

f $x = 0 \ \therefore \ y = -\frac{13}{10}$

$\frac{dy}{dx} = \frac{1}{10} - \frac{3}{10}e^x$, grad $= -\frac{1}{5}$

$\therefore$ grad of normal $= 5$

$\therefore \ y = 5x - \frac{13}{10}$