DIFFERENTIATION

1 Differentiate with respect to x

    a e^x **b** $3e^x$ **c** $\ln x$ **d** $\frac{1}{2}\ln x$

2 Differentiate with respect to t

    a $7 - 2e^t$ **b** $3t^2 + \ln t$ **c** $e^t + t^5$ **d** $t^{\frac{3}{2}} + 2e^t$

    e $2\ln t + \sqrt{t}$ **f** $2.5e^t - 3.5\ln t$ **g** $\frac{1}{t} + 8\ln t$ **h** $7t^2 - 2t + 4e^t$

3 Find $\dfrac{d^2y}{dx^2}$ for each of the following.

    a $y = 4x^3 + e^x$ **b** $y = 7e^x - 5x^2 + 3x$ **c** $y = \ln x + x^{\frac{5}{2}}$

    d $y = 5e^x + 6\ln x$ **e** $y = \dfrac{3}{x} + 3\ln x$ **f** $y = 4\sqrt{x} + \frac{1}{4}\ln x$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

    a $f(x) = 3x + e^x$, $x = 0$ **b** $f(x) = \ln x - x^2$, $x = 4$

    c $f(x) = x^{\frac{1}{2}} + 2\ln x$, $x = 9$ **d** $f(x) = 5e^x + \dfrac{1}{x^2}$, $x = -\frac{1}{2}$

5 Find, in each case, any values of x for which $\dfrac{dy}{dx} = 0$.

    a $y = 5\ln x - 8x$ **b** $y = 2.4e^x - 3.6x$ **c** $y = 3x^2 - 14x + 4\ln x$

6 Find the value of x for which $f'(x)$ takes the value indicated in each case.

    a $f(x) = 2e^x - 3x$, $f'(x) = 7$ **b** $f(x) = 15x + \ln x$, $f'(x) = 23$

    c $f(x) = \dfrac{x^2}{8} - 2x + \ln x$, $f'(x) = -1$ **d** $f(x) = 30\ln x - x^2$, $f'(x) = 4$

7 Find the coordinates and the nature of any stationary points on each of the following curves.

    a $y = e^x - 2x$ **b** $y = \ln x - 10x$ **c** $y = 2\ln x - \sqrt{x}$

    d $y = 4x - 5e^x$ **e** $y = 7 + 2x - 4\ln x$ **f** $y = x^2 - 26x + 72\ln x$

8 Given that $y = x + ke^x$, where k is a constant, show that

$$(1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

9 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

    a $y = e^x$, $x = 2$ **b** $y = \ln x$, $x = 3$ **c** $y = 0.8x - 2e^x$, $x = 0$

    d $y = 5\ln x + \dfrac{4}{x}$, $x = 1$ **e** $y = x^{\frac{1}{3}} - 3e^x$, $x = 1$ **f** $y = \ln x - \sqrt{x}$, $x = 9$

10 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate.

    a $y = \ln x$, $x = e$ **b** $y = 4 + 3e^x$, $x = 0$ **c** $y = 10 + \ln x$, $x = 3$

    d $y = 3\ln x - 2x$, $x = 1$ **e** $y = x^2 + 8\ln x$, $x = 1$ **f** $y = \frac{1}{10}x - \frac{3}{10}e^x - 1$, $x = 0$