DIFFERENTIATION Answers

1 **a** **i** $\frac{dy}{dx} = 2x + 3$

ii $\frac{dx}{dt} = 3(t-4)^2$

b **i** $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$t = 5,\ \frac{dx}{dt} = 3,\ x = 1,\ \frac{dy}{dx} = 5$

$\therefore\ \frac{dy}{dt} = 5 \times 3 = 15$

ii $x = 8,\ \frac{dy}{dx} = 19$

$8 = (t-4)^3\ \therefore\ t = 6,\ \frac{dx}{dt} = 12$

$\therefore\ \frac{dy}{dt} = 19 \times 12 = 228$

2 $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dx} = 1 \times \sqrt{2x-3} + x \times \frac{1}{2}(2x-3)^{-\frac{1}{2}} \times 2$

$= \frac{(2x-3)+x}{\sqrt{2x-3}} = \frac{3(x-1)}{\sqrt{2x-3}}$

$x = 6,\ \frac{dx}{dt} = 0.3,\ \frac{dy}{dx} = 5$

$\therefore\ \frac{dy}{dt} = 5 \times 0.3 = 1.5$

$\therefore$ y increasing at 1.5 units per second

3 **a** $\frac{dP}{dt} = \frac{dP}{dr} \times \frac{dr}{dt}$

$\frac{dr}{dt} = 0.2,\ P = 2\pi r\ \therefore\ \frac{dP}{dr} = 2\pi$

$\therefore\ \frac{dP}{dt} = 2\pi \times 0.2 = 0.4\pi$

$\therefore$ perimeter increasing at 0.4π cm s^{-1}

b $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$A = \pi r^2\ \therefore\ \frac{dA}{dr} = 2\pi r$

$r = 10,\ \frac{dA}{dr} = 20\pi$

$\therefore\ \frac{dA}{dt} = 20\pi \times 0.2 = 4\pi$ cm^2 s^{-1}

c $20 = 2\pi r \times 0.2\ \therefore\ r = \frac{50}{\pi} = 15.9$ cm (3sf)

4 **a** $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$A = \pi r^2\ \therefore\ \frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = -0.5,\ r = 8\ \therefore\ \frac{dA}{dr} = 16\pi$

$\therefore\ -0.5 = 16\pi \times \frac{dr}{dt}$

$\frac{dr}{dt} = -\frac{1}{32\pi} = -0.00995$ cm s^{-1}

$\therefore$ radius decreasing at 0.00995 cm s^{-1} (3sf)

b $\frac{dP}{dt} = \frac{dP}{dr} \times \frac{dr}{dt}$

$P = 2\pi r\ \therefore\ \frac{dP}{dr} = 2\pi$

$\therefore\ \frac{dP}{dt} = 2\pi \times -\frac{1}{32\pi} = -\frac{1}{16}$

$\therefore$ perimeter decreasing at 0.0625 cm s^{-1}

5 **a** $\frac{dV}{dt} = \frac{dV}{dl} \times \frac{dl}{dt}$

$\frac{dV}{dt} = 3.5,\ V = l^3 \ \therefore\ \frac{dV}{dl} = 3l^2$

$200 = l^3 \ \therefore\ l = \sqrt[3]{200} = 5.848$

$\therefore\ \frac{dV}{dl} = 3 \times (5.848)^2 = 102.6$

$\therefore\ 3.5 = 102.6 \times \frac{dl}{dt}$

$\frac{dl}{dt} = 3.5 \div 102.6 = 0.0341 \text{ cm s}^{-1}$

b $2 \text{ mm s}^{-1} = 0.2 \text{ cm s}^{-1}$

$3.5 = \frac{dV}{dl} \times 0.2$

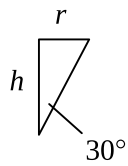
$\therefore\ \frac{dV}{dl} = 3.5 \div 0.2 = 17.5$

$\therefore\ 17.5 = 3l^2$

$l = \sqrt{\frac{17.5}{3}} = 2.415$

$V = (2.415)^3 = 14.1 \text{ cm}^3$ (3sf)

6 **a**



$\tan 30° = \frac{1}{\sqrt{3}} = \frac{r}{h}$

$\therefore\ r = \frac{h}{\sqrt{3}}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \frac{1}{3}h^2 \times h = \frac{1}{9}\pi h^3$

b $t = 120,\ V = 565.06$

$\therefore\ h = \sqrt[3]{\frac{9 \times 565.06}{\pi}} = 11.7$ cm (3sf)

c $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$\frac{dV}{dh} = \frac{1}{3}\pi h^2$

$h = 11.74,\ \frac{dV}{dh} = 144.4$

$\frac{dV}{dt} = 600 \times (-0.0005)e^{-0.0005t}$

$= -0.3e^{-0.0005t}$

$t = 120,\ \frac{dV}{dt} = -0.2825$

$\therefore\ -0.2825 = 144.4 \times \frac{dh}{dt}$

$\frac{dh}{dt} = -0.00196$

$\therefore$ depth decreasing at $0.00196 \text{ cm s}^{-1}$ (3sf)