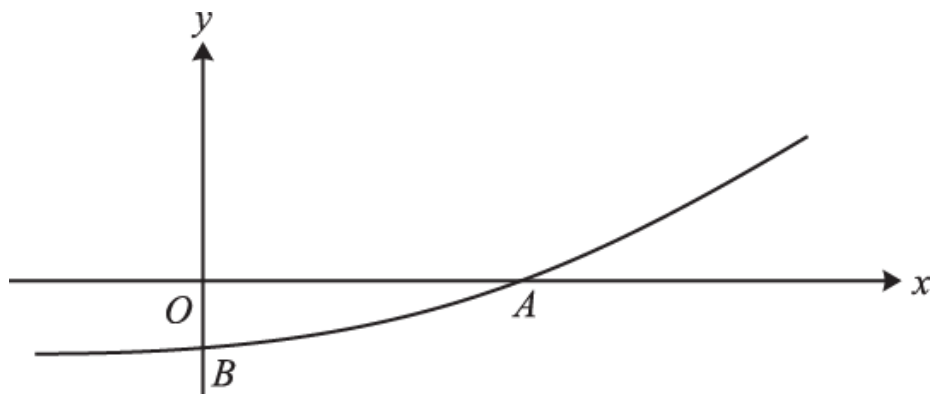


1.



The diagram shows the curve with equation

$$x = (y + 4)\ln(2y + 3).$$

The curve crosses the x -axis at A and the y -axis at B .

- i. Find an expression for $\frac{dx}{dy}$ in terms of y .

[3]

- ii. Find the gradient of the curve at each of the points A and B , giving each answer correct to 2 decimal places.

[5]

2. For each of the following curves, find the gradient at the point with x -coordinate 2.

i. $y = \frac{3x}{2x + 1}$

[3]

ii. $y = \sqrt{4x^2 + 9}$

[3]

3. Find the exact value of the gradient of the curve

$$y = \sqrt{4x - 7} + \frac{4x}{2x + 1}$$

at the point for which $x = 4$.

[6]

4. The functions f and g are defined for all real values of x by

$$f(x) = 2x^3 + 4 \quad \text{and} \quad g(x) = \sqrt[3]{x-10}.$$

i. Evaluate $f^{-1}(-50)$.

[2]

ii. Show that $fg(x) = 2x - 16$.

[2]

iii. Differentiate $gf(x)$ with respect to x .

[3]

5.

Given that $y = 4x^2 \ln x$, find the value of $\frac{d^2y}{dx^2}$ when $x = e^2$.

[5]

6.

Find the equation of the tangent to the curve $y = \frac{5x+4}{3x-8}$ at the point $(2, -7)$.

[5]

7. The curves C_1 and C_2 have equations

$$y = \ln(4x - 7) + 18 \quad \text{and} \quad y = a(x^2 + b)^{\frac{1}{2}}$$

respectively, where a and b are positive constants. The point P lies on both curves and has x -coordinate 2. It is given that the gradient of C_1 at P is equal to the gradient of C_2 at P . Find the values of a and b .

[8]

8. Find the equation of the tangent to the curve

$$y = 3x^2(x+2)^6$$

at the point $(-1, 3)$, giving your answer in the form $y = mx + c$.

[5]

9. The equation of a curve is $y = e^{2x} \cos x$. Find $\frac{dy}{dx}$ and hence find the coordinates of any stationary points for which $-\pi \leq x \leq \pi$. Give your answers correct to 3 significant figures. [6]

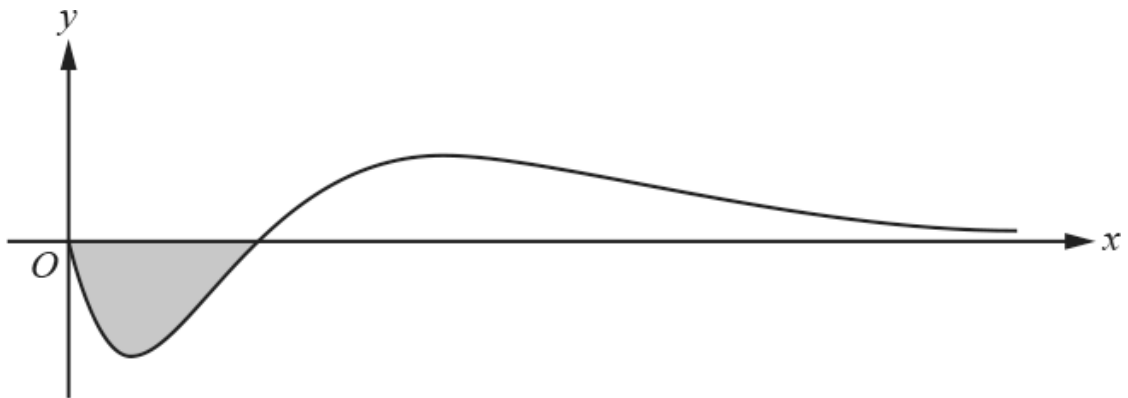
10. A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant. Given that the curve has a minimum point when $x = -2$

- find the value of k ,
- show that the curve has a point of inflection which is not a stationary point.

[7]

11. The equation of a curve has the form $y = e^{x^2} (ax^2 + b)$, where a and b are non-zero constants. It is given that $\frac{d^2y}{dx^2}$ can be expressed in the form $e^{x^2} (cx^4 + d)$, where c and d are non-zero constants. Prove that $5a + 2b = 0$. [5]

12. In this question you must show detailed reasoning.



The function f is defined for the domain $x \geq 0$ by

$$f(x) = (2x^2 - 3x)e^{-x}.$$

The diagram shows the curve $y = f(x)$.

(a) Find the range of f . [6]

- (b) The function g is defined for the domain $x \geq k$ by

$$g(x) = (2x^2 - 3x)e^{-x}.$$

Given that g is a one-one function, state the least possible value of k . [1]

- (c) Find the exact area of the shaded region enclosed by the curve and the x -axis. [7]

13. In this question you must show detailed reasoning.

A curve has equation $y = \frac{\ln x}{x}$.

- (a) Find the x -coordinate of the point where the curve crosses the x axis. [2]
- (b) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x -axis. [2]
- (c) Find the coordinates of the turning point on the curve. [4]
- (d) Determine whether this turning point is a maximum or a minimum. [5]

- 14.

A function f is defined for $x > 0$ by $f(x) = \frac{6}{x^2 + a}$, where a is a positive constant.

- (a) Show that f is a decreasing function. [4]
- (b) Find, in terms of a , the coordinates of the point of inflection on the curve $y = f(x)$. [5]

END OF QUESTION paper

Mark scheme

Question	Answer/Indicative content	Marks	Part marks and guidance
1	<p data-bbox="181 363 465 391">i Attempt use of product rule</p> <p data-bbox="181 515 394 542">i Obtain $\ln(2y + 3) \dots$</p> <p data-bbox="181 943 584 1038">i Obtain $\dots + \frac{2(y + 4)}{2y + 3}$</p>	<p data-bbox="994 363 1032 391">M1</p> <p data-bbox="994 515 1032 542">A1</p> <p data-bbox="994 986 1032 1013">A1</p>	<p data-bbox="1093 284 1648 343">to produce expression of form (something non-zero) $\ln(2y + 3) +$ <u>linear in y</u></p> <p data-bbox="1093 355 1323 399"><u>linear in y</u> ignore</p> <p data-bbox="1093 448 1339 475">what they call their derivative</p> <p data-bbox="1093 515 1285 542">with brackets included</p> <p data-bbox="1093 582 1404 609">with brackets included as necessary</p> <p data-bbox="1093 655 1301 683"><u>Examiner's Comments</u></p> <p data-bbox="1093 727 1805 786">A few candidates did not recognise the need to use the product rule here but most did and, indeed, 58% of candidates duly earned all three marks. A common error</p> <p data-bbox="1093 786 1814 919">was the differentiation of $\ln(2y + 3)$ to produce $\frac{1}{2y + 3}$ and, in many cases, one of</p> <p data-bbox="1093 967 1792 1026">the terms was $\ln(2y + 3)$. Full credit was not given when necessary brackets were absent; $\ln 2y + 3$ appeared in many answers. As soon as a correct expression</p> <p data-bbox="1093 1026 1442 1126">for $\frac{dx}{dy}$ was produced, the marks were</p> <p data-bbox="1093 1174 1805 1233">awarded. This was fortunate for many candidates as some subsequent horrendous 'simplification' was perpetrated, including the</p> <p data-bbox="1093 1233 1630 1329">correct $\frac{dx}{dy} = \ln(2y + 3) + \frac{2y + 8}{2y + 3}$ becoming</p> <p data-bbox="1093 1329 1312 1414">$\ln(2y + 3) + \frac{8}{3}$.</p>

	ii	Substitute $y = 0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
	ii	Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558...; beware of 'correct' answer coming from incorrect version $\ln(2y + 3) + \frac{8}{3}$ of answer in part (i)
	ii	Attempt to find value of y for which $x = 0$	M1	allowing process leading only to $y = -4$
	ii	Substitute $y = -1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	or greater accuracy 0.16666...; value following from correct working
				<p>Examiner's Comments</p> <p>This question assessed the specification item 'understand and use the relation $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$'. It was not answered well in</p> <p>general and only 13% of candidates were able to earn all five marks. There were several major problems as far as candidates were concerned. Many candidates thought that the gradients could be found by substitution into the expression from part (i); many therefore claimed the gradient at A as 3.77 and rather fewer decided on 6 as the gradient at B.</p> <p style="text-align: right;">$\frac{dy}{dx}$ $\frac{dx}{dy}$</p> <p>Other candidates, with a little awareness of a difference between $\frac{dy}{dx}$ and $\frac{dx}{dy}$, decided</p> <p>that all would be well if x and y were interchanged throughout and relevant x-values substituted into their adjusted expression from part (i). Another problem arose for</p> <p style="text-align: center;">$\frac{dy}{dx}$</p> <p>those candidates trying to produce an expression for $\frac{dy}{dx}$. There may</p> <p>already have been some 'simplification' as mentioned above but, for those still with</p>
ii		Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	

				$\frac{dx}{dy}$ <p>a correct expression for $\frac{dx}{dy}$, there was the</p> $\frac{dx}{dy}$ <p>problem of finding its reciprocal. A few did this correctly after expressing $\frac{dx}{dy}$ with a</p> <p>common denominator but, for far too</p> $\frac{dx}{dy} = \ln(2y+3) + \frac{2y+8}{2y+3}$ <p>many, $\frac{dx}{dy} = \frac{1}{\ln(2y+3)} + \frac{2y+3}{2y+3}$ became</p> $\frac{dx}{dy} = \frac{1}{\ln(2y+3)} + \frac{2y+3}{2y+8}$ <p>A final problem</p> <p>concerned the values of y to be substituted. Most realised that $y=0$ was appropriate for the point A but finding the y-value for B required more thought which, commendably, some candidates did display with a comment about $y=-4$ being impossible because it led to the logarithm of a negative number.</p> <p>Successful candidates usually avoided all the algebraic problems by substituting</p> $\frac{dx}{dy}$ <p>each of the two y-values into the expression for $\frac{dx}{dy}$ and then finding the reciprocal of each numerical value. This uncomplicated approach was not seen very often.</p>
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		Total	8	
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2	i	<p>Either Attempt use of quotient rule</p> $\frac{3(2x+1) - 6x}{(2x+1)^2}$ <p>Obtain or equiv</p>	<p>M1</p> <p>A1</p>	<p>allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving x; for M1 condone minor errors such as absence of square in denominator, absence of brackets, ...</p> <p>give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'</p>
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		<p>i Substitute 2 to obtain $\frac{3}{25}$ or 0.12</p> <p>i Or Attempt use of product rule for $3x(2x+1)^{-1}$</p> <p>i Obtain $3(2x+1)^{-1} - 6x(2x+1)^{-2}$ or equiv</p> <p>i Substitute 2 to obtain $\frac{3}{25}$ or 0.12</p>	<p>A1 or simplified equiv but A0 for final $\frac{3}{5^2}$</p> <p>M1 allow sign error; condone no use of chain rule</p> <p>A1</p> <p>or simplified equiv</p> <p>Examiner's Comments</p> <p>This question was answered well with 75% of candidates earning all three marks. Use of the quotient rule was the usual approach; a few candidates had the terms in the numerator the wrong way round but a more common error, and an avoidable one, was the simplification of $3(2x+1) - 6x$ in the numerator to give 1. Some candidates opted for the product rule and were not always successful, failure to apply the chain rule being the principal cause of error.</p>	
		<p>ii Differentiate to obtain form $kx(4x^2+9)^n$</p> <p>ii Obtain $4x(4x^2+9)^{-\frac{1}{2}}$</p> <p>ii Substitute 2 to obtain $\frac{8}{5}$ or 1.6</p>	<p>M1 any non-zero constants k and n (including 1 or $\frac{1}{2}$ or n)</p> <p>A1 or (unsimplified) equiv</p> <p>or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$</p> <p>Examiner's Comments</p> <p>This was also answered well, again with 75% of candidates earning three marks. Failure to include a factor x in the derivative was the most common error. In this part, and in part (i), a number of candidates omitted to substitute 2 to find the gradient as requested.</p>	
		Total	6	
3		Differentiate first term to obtain form $k(4x-7)^{-\frac{1}{2}}$	*M1	any non-zero constant k ; M0 if this differentiation is carried out in the midst of some incorrect involved expression

		<p>Obtain $2(4x - 7)^{-\frac{1}{2}}$</p> <p>Attempt use of quotient rule or, after adjustment, product rule</p> $\frac{4(2x + 1) - 8x}{(2x + 1)^2} \quad \text{or } 4(2x + 1)^{-1} - 8x(2x + 1)^{-2}$ <p>Substitute 4 into expression for first derivative so that (initially at least) exactness is retained</p> <p>Obtain $\frac{58}{81}$</p>	<p>A1 or (unsimplified) equiv</p> <p>*M1 for QR, allow numerator wrong way round but needs – sign in numerator; condone a single error such as absence of square in denominator, absence of brackets, ...; for PR, condone no use of chain rule M0 if this differentiation is carried out in the midst of some incorrect involved expression</p> <p>A1 or (unsimplified) equivs; give A0 if brackets absent unless subsequent calculation indicates their 'presence'</p> <p>M1 dep *M *M</p> <p>answer must be exact</p> $y = \sqrt{4x - 7} + \frac{4}{2x + 1}$ <p>Note: using do not apply MR</p> <p>Examiner's Comments</p> <p>A1 This question was answered very well and 63% of candidates recorded all 6 marks. The first term was usually differentiated correctly but there were a few more problems with the second term. Careless simplification often led to an expression $\frac{1}{(2x + 1)^2}$ for those candidates using the quotient rule. Some candidates rewrote the expression as $4x(2x + 1)^{-1}$; a few did not use the product rule and, for some others, there were errors as the chain rule was not used. The vast majority of candidates recognised the need to give an exact answer and there were few instances where candidates resorted to decimal approximations.</p>	
		Total	6	
4	i	Either: State $2x^8 + 4 = -50$	B1	
	i	State -3 and no other	B1	Examiner's Comments

				<p>There were few problems with part (i) and 83% of candidates earned both marks. Few seemed to realise that the answer can be obtained by solving $f(x) = -50$ and the common approach was to find the inverse function. Many were guilty of careless notation, writing the inverse in a way to suggest $\frac{\sqrt[3]{x-4}}{2}$ when they clearly meant $\sqrt[3]{\frac{x-4}{2}}$;</p> <p>provided they proceeded to carry out the correct calculation, both marks were earned. Further carelessness was evident on some scripts where 50 was substituted.</p>	
	i	Or: Obtain $\sqrt[3]{\frac{1}{2}(x-4)}$ for inverse of f	B1	or equiv; using any letter	
	i	State -3 and no other	B1		
	ii	Show composition of functions the right way round	M1	AG; necessary detail needed	
	ii	Obtain $2x - 16$	A1	<p>Examiner's Comments</p> <p>Part (ii) was answered extremely well with almost all candidates showing sufficient detail and recording two marks.</p>	first step $2(x - 10) + 4$ acceptable but then two more steps needed
	iii	Obtain $\sqrt[3]{2x^3 - 6}$ or $(2x^3 - 6)^{\frac{1}{3}}$ for gf (x)	B1	or unsimplified equiv	
	iii	Apply chain rule to function which is cube root of a non-linear expression	M1	condone incorrect constant; otherwise use of chain rule for their function must be correct	may use $u = 2x^3 - 6$; M1 earned for expression involving u
	iii	Obtain $x = \sqrt[4]{9 + 8x - x^2}$	A1	<p>or similarly simplified equiv; do not accept final answer with $\frac{6}{3}$ unsimplified</p> <p>Examiner's Comments</p>	... in terms of x

				<p>There were more problems with part (iii). Almost all candidates had the correct expression for gf (x) but some chose not to carry out the obvious simplification, or simplified it to $\sqrt[3]{-20x^3 - 40}$ or to $(2x^3 + 4)^{\frac{1}{3}} - 10^{\frac{1}{3}}$. Common errors with the differentiation included a factor 6x instead of 6x², an expression involving $(2x^3 - 6)^{-\frac{1}{3}}$ and a final answer not suitably simplified.</p>
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		Total	7	
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5	<p>Attempt use of product rule to find first derivative</p> <p>Obtain $8x \ln x + 4x$</p> <p>Attempt use of correct product rule to find second derivative</p> <p>Obtain $8 \ln x + 12$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>producing form ... ± ... where one term involves ln x and the other does not</p> <p>or unsimplified equiv</p> <p>with one term involving ln x</p> <p>or unsimplified equiv</p> <p>Examiner's Comments</p> <p>This question was a suitable introduction to the paper for the majority of candidates, and 77% of them duly earned all five marks. Some provided very concise solutions taking only a few lines of working; the two applications of the product rule were handled without fuss. For many other candidates, solutions were more protracted with each attempt at the product rule needing some work at the side as functions u and v were defined and differentiated. Assembling the parts to form each derivative was prone to error and one that occurred frequently was a failure to include the derivative of 4x in the expression for the second derivative. Sound advice for candidates setting out solutions is to carry out obvious simplifications as the solution progresses. It was surprising that a significant number of candidates did not do this in this question. Having found the first derivative as $8x \ln x + \frac{4x^2}{x}$, they continued</p> <p>by correctly applying the product rule to the first term and then using the quotient</p>
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				rule to deal with the second term. A few candidates did not see the need to use the product rule at all and the first step in a few cases was the substitution of e^2 .	
		Obtain 28	A1		
		Total	5		
6		<p>Attempt use of quotient rule or, after adjustment, product rule</p> <p>Obtain</p> $\frac{5(3x - 8) - 3(5x + 4)}{(3x - 8)^2}$ <p>or</p> <p>equiv</p> <p>Substitute 2 to obtain -13 or equiv</p> <p>Attempt to find equation of tangent</p> <p>Obtain $y = -13x + 19$ or $13x + y - 19 = 0$</p>	<p>*M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x - 8$ in denominator; allow M1 for numerator the wrong way round</p> <p>Allow if missing brackets implied by subsequent simplification or calculation</p> <p>Dep *M; equation of tangent not normal</p> <p>Or similarly simplified equiv with 3 non-zero terms</p> <p>Examiner's Comments</p> <p>This opening question was answered very well in general with 74% of the candidates recording full marks. The majority applied the quotient rule accurately although lack of care with brackets in the numerator did lead to some sign errors. Some candidates opted for use of the product rule and this was not handled quite so convincingly. There were some cases where candidates stopped as soon as they had found the gradient but, in general, candidates proceeded without difficulty to produce the equation of the tangent and to present it in an acceptable form.</p>	<p>For product rule attempt, *M1 for $k_1(3x - 8)^{-1} + k_2(5x + 4)(3x - 8)^{-2}$ form and A1 for correct constants 5 and -3;</p>
		Total	5		
7		State, at some stage, $a(4 + b)\frac{1}{2} = 18$	B1		

	<p>Obtain derivative $\frac{4}{4x-7}$ for C_1</p> <p>Obtain derivative $kx(x^2 + b)^{-\frac{1}{2}}$ for C_2</p> <p>Obtain correct $ax(x^2 + b)^{-\frac{1}{2}}$</p> <p>Equate derivatives with $x = 2$</p> <p>Attempt values of a and b from two equations involving a and $(4 + b)^{-\frac{1}{2}}$</p> <p>Obtain $a = 6$</p> <p>Obtain $b = 5$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Any non-zero constant k</p> <p>Using correct process</p> <p>Correct equations are $a(4 + b)^{-\frac{1}{2}} = 18$ and $2a(4 + b)^{-\frac{1}{2}} = 4$</p> <p>Examiner's Comments</p> <p>Responses to this question varied considerably. Some candidates proceeded logically and efficiently and were able to produce a correct solution in half a page. For other candidates, there seemed to be no plan and solutions meandered on for several pages of complicated algebra. Careful reading of the question was essential and a major problem on many scripts was the failure to form the second equation $18 = a(4 + b)^{-\frac{1}{2}}$ resulting from the fact that the two curves meet where $x = 2$. Differentiation was often not correct, with the derivative of C_1 appearing as $\frac{1}{4x-7}$ a common error.</p> <p>Candidates who had differentiated correctly and also established the second equation were faced with the equations $18 = a(4 + b)^{-\frac{1}{2}}$ and $2a(4 + b)^{-\frac{1}{2}} = 4$. Some candidates saw that it was then a straightforward process to eliminate one of a and b; they proceeded to the correct values without fuss. Many other candidates dealt with these equations by squaring both sides of both equations, leading them into quadratic equations with large coefficients and increasing the chances of careless slips. Algebraic errors such as $(4 + b)^{-\frac{1}{2}} = 2 +$</p>
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$\frac{1}{b}$ occurred far too often for an examination at this level. Some candidates did not help themselves by delaying the substitution of $x = 2$ until a late stage of their solutions.

Full marks were earned by 45% of the candidates but the solutions of other candidates did reveal uncertainties with the necessary algebra and calculus techniques.

8	<p>Differentiate to produce form</p> $k_1x(x+2)^m + k_2x^2(x+2)^n$ <p>Obtain $6k_1x(x+2)^6 + 18k_2x^2(x+2)^5$</p> <p>Substitute $x = -1$ to obtain value 12</p> <p>Attempt equation of tangent (not normal) through point $(-1, 3)$</p> <p>Obtain $y = 12x + 15$</p>	<p>*M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For positive integers k_1, k_2, m, n; allow M1 if slip to, for example, $(x + 3)$ in both brackets</p> <p>Or unsimplified equiv</p> <p>From correct work only</p> <p>Dep *M; using non-zero numerical value of gradient; condone slip in use of coordinates</p> <p>Answer required in $y = mx + c$ form</p> <p>Examiner's Comments</p> <p>Most candidates found this a straightforward opening question and 80% of the candidates duly recorded full marks. The product rule was used efficiently by all but a few candidates. There were some careless errors when substituting -1 in the derivative and a few more as candidates manipulated the equation to the requested form. There were very few instances of candidates using a gradient of $-\frac{1}{12}$ and, in effect, finding the equation of the normal.</p>
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9	$\frac{dy}{dx} = \pm ke^{2x} \cos x \pm e^{2x} \sin x$	M1*	<p>k is any constant</p>	Product Rule
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$$\frac{dy}{dx} = 2e^{2x} \cos x - e^{2x} \sin x \text{ oe}$$

$$\frac{dy}{dx} = 0$$

their

$\tan x = 2$ or

$$\cos x = (\pm) \frac{1}{\sqrt{5}} \text{ or } \sin x = (\pm) \frac{2}{\sqrt{5}}$$

$x = 1.11$ and -2.03 cao

$y = 4.09$ and -0.00765 cao

A1

M1dep*

A1

M1

A1

ignore omission of " $e^{2x} = 0$ has no solution"

(1.11, 4.09) and / or (-2.03, -0.00765)

or **A1** for each correct pair of co-ordinates: mark to benefit of candidate

extra values within range incur a penalty of one mark;

or

any finite value for x obtained from $e^{2x} = 0$

incurs a penalty of one mark

Examiner's Comments

The differentiation was very well done by nearly all candidates, and an overwhelming majority set the derivative equal to zero and successfully identified $\tan x = 2$.

Thereafter many lost accuracy or omitted either the y - values or one of the x -value.

Only a few candidates found incorrect finite values from $e^{2x} = 0$, rather more failed to recognise that $\tan x$ was available, and worked with $\sin^2 x$ or $\cos^2 x$, thus nearly always introducing incorrect extra values in the specified range.

A very small number of candidates integrated instead of differentiating.

$$\text{or } \sqrt{5} \cos(x + \tan^{-1} \frac{1}{2}) = 0$$

if **A0A0, SC1** for all 4 values to greater precision 1.107..., -2.034..., 4.094..., -0.0076457... (or -0.007646)

NB

$x = 1.107148718$ and -2.034443936
 $y = 4.094229238$ and -0.007645738

ignore extra values outside range

			Total	6		
10		$\frac{dy}{dx} = 2x + k + 4x^{-2}$ $2(-2) + k + 4(-2)^{-2} = 0$ $k = 3$ $\frac{d^2y}{dx^2} = 2 - 8x^{-3}$ $2 - 8x^{-3} = 0$ $x = 4^{\frac{1}{3}}$ <p>for $x < 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} < 0$</p> <p>for $x > 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} > 0$</p> <p>when $x = 4^{\frac{1}{3}}, \frac{dy}{dx} \neq 0$ hence not a stationary point</p>	M1(AO 1.1a) M1(AO3.1a) A1(AO1.1) M1(AO3.1a) A1(AO1.1) E1(AO2.1) E1(AO2.1) [7]	Attempt to differentiate Substitute $x = -2$, equate to 0 and attempt to solve Equate second derivative to 0 and attempt to solve Consider convex / concave either side of $x = 4^{\frac{1}{3}}$ and conclude Consider gradient at $x = 4^{\frac{1}{3}}$, or justify that x $= -2$ is the only stationary point	Power decreases by 1 for at least 2 terms	
			Total	7		
11		Differentiate to obtain form $e^{ax} (px^2 + qx)$		M1		

		$\frac{dy}{dx} = 2xe^{x^2}(ax^2 + b) + 2axe^{x^2}$ <p>Obtain</p> $\frac{d^2y}{dx^2} = e^{x^2}(4ax^4 + 10ax^2 + 4bx^2 + 2a + 2b)$ <p>Obtain</p> <p>Equate coefficient of $x^2e^{x^2}$ to zero</p> <p>Confirm $5a + 2b = 0$</p>	<p>A1</p> <p>Or equiv</p> <p>A1</p> <p>Or equiv</p> <p>M1</p> <p>Provided second derivative involves $e^{x^2}x^4$, $e^{x^2}x^2$ and e^{x^2} terms and no others</p> <p>A1</p> <p>AG – necessary detail needed</p> <p>Examiner's Comments</p> <p>[5]</p> <p>A significant number of candidates made no progress with part (b) and some were unable to devise a strategy. Many others did earn the first two marks with accurate work in finding the first derivative. Indeed it was pleasing that many were able to differentiate the awkward e^{x^2} correctly. Further success usually depended on candidates organising their first derivative into a form suitable for further differentiation. Candidates who tried to differentiate a term such as $2xe^{x^2}(ax^2 + b)$ occasionally succeeded but generally were unable to cope, either failing to use the product rule appropriately or making careless slips. Candidates who organised their first derivative into a form such as $e^{x^2}(2ax^2 + 2ax + 2bx)$ were then faced with a manageable task to find the second derivative. Some of the candidates who reached an expression for the second derivative did not realise the implication provided by the form of the second derivative given in the question and were unable to conclude successfully. But it is pleasing to record the fact that approximately one-fifth of the candidates did manage to prove the result.</p>				
		Total	9				
12	a	<p>DR</p> <p>Attempt product rule for y</p> <p>$y' = (4x - 3)e^{-x} - (2x^2 - 3x)e^{-x}$</p>	<p>M1 (AO 3.1a)</p>	<table border="1"> <tr> <td> <p>Attempt must be of the form</p> $(ax + b)e^{-x} \pm (cx^2 + dx)e^{-x}$ </td> <td></td> </tr> </table>	<p>Attempt must be of the form</p> $(ax + b)e^{-x} \pm (cx^2 + dx)e^{-x}$		
<p>Attempt must be of the form</p> $(ax + b)e^{-x} \pm (cx^2 + dx)e^{-x}$							

		$y' = 0 \Rightarrow (4x - 3) - (2x^2 - 3x) = 0$ <p>Obtain quadratic in x and attempt to solve</p> $x = \frac{1}{2}, \quad x = 3$ $-e^{-\frac{1}{2}} \leq y \leq 9e^{-3}$	<p>A1 (AO 1.1)</p> <p>M1 (AO 2.1)</p> <p>M1 (AO 1.1)</p> <p>A1 (AO 1.1)</p> <p>A1 (AO 2.5)</p> <p>[6]</p>	<p>Correct derivative, in any form</p> <p>Set $y' = 0$ and eliminate exponentials</p> <p>Dependent on both previous M marks</p> <p>Correct values from correct equation</p> <p>Correct range, including correct inequality signs and either y, f or $f(x)$ used for range notation (not x)</p>	$2x^2 - 7x + 3 = 0$ <p>Allow 'closed interval' notation</p> $[-e^{-\frac{1}{2}}, 9e^{-3}]$	
	b	DR $k = 3$	<p>B1ft (AO 2.3)</p> <p>[1]</p>	<p>FT their larger value of x from (a)</p>		
	c	<p>Use integration by parts with $u = 2x^2 - 3x$ and $v' = e^{-x}$</p> $\int (2x^2 - 3x)e^{-x} dx = -(2x^2 - 3x)e^{-x} + \int (4x - 3)e^{-x} dx$ <p>Attempt parts again with $u = ax + b$ and $v' = e^{-x}$</p> $\int (2x^2 - 3x)e^{-x} dx = -(2x^2 + x + 1)e^{-x} + c$	<p>M1 (AO 1.1)</p> <p>A1 (AO 1.1)</p> <p>M1 (AO 1.1)</p> <p>A1 (AO 1.1)</p>	<p>Must obtain result $f(x) \pm \int g(x) dx$</p> <p>Dependent on previous M mark</p>		

		$2x^2 - 3x = 0 \Rightarrow x = \frac{3}{2} \quad (\text{and } x = 0)$ <p>Correct use of correct limits</p> $\text{Integral is } 1 - 7e^{-\frac{3}{2}} < 0 \text{ so area is } 7e^{-\frac{3}{2}} - 1$	<p>B1 (AO 3.1a)</p> <p>M1 (AO 1.1)</p> <p>A1 (AO 2.2a)</p> <p>[7]</p>	<p>oe; accept unsimplified (but all bracketing must be correct)</p> <p>Dependent on both previous M marks</p>										
		Total	14											
13	a	<p>DR</p> $\frac{\ln x}{x} = 0$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; height: 40px;"></td> <td style="width: 33%; text-align: center; vertical-align: middle;"> $\frac{\ln 1}{1} = 0$ or </td> <td style="width: 33%; height: 40px;"></td> </tr> </table> <p>$\Rightarrow \ln x = 0$</p> <p>$\Rightarrow x = 1$</p>		$\frac{\ln 1}{1} = 0$ or		<p>M1 (AO 1.1a)</p> <p>A1 (AO 1.1)</p> <p>[2]</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; height: 100px; vertical-align: top;">May not be seen</td> <td style="width: 50%; height: 100px;"></td> </tr> </table> <p>May be implied</p> <p><u>Examiner's Comments</u></p> <p>This question was well answered.</p>	May not be seen						
	$\frac{\ln 1}{1} = 0$ or													
May not be seen														
	b	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20%;">y-coordinates are</td> <td style="width: 15%; text-align: center;">$\frac{\ln 2}{2}$</td> <td style="width: 10%; text-align: center;">and</td> <td style="width: 15%; text-align: center;">$\frac{\ln 4}{4}$</td> <td style="width: 40%;"></td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr> <td style="width: 70%;">$\frac{\ln 4}{4} = \frac{2 \ln 2}{4} = \frac{\ln 2}{2}$</td> <td style="width: 30%;">oe</td> </tr> </table>	y-coordinates are	$\frac{\ln 2}{2}$	and	$\frac{\ln 4}{4}$		$\frac{\ln 4}{4} = \frac{2 \ln 2}{4} = \frac{\ln 2}{2}$	oe	<p>B1* (AO 1.1)</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; height: 100px; vertical-align: top;">Allow</td> <td style="width: 50%; height: 100px; vertical-align: top;">Both = 0.346... BOB0</td> </tr> </table>	Allow	Both = 0.346... BOB0	
y-coordinates are	$\frac{\ln 2}{2}$	and	$\frac{\ln 4}{4}$											
$\frac{\ln 4}{4} = \frac{2 \ln 2}{4} = \frac{\ln 2}{2}$	oe													
Allow	Both = 0.346... BOB0													

⇒ AB is / to x-axis AG

B1dep*
(AO 3.1a)

[2]

$$\frac{\ln 4}{4} = \ln 4^{\frac{1}{4}} = \ln \sqrt{2} = \frac{\ln 2}{2}$$

use of
 $\frac{\ln 4}{4} - \frac{\ln 2}{2} = 0$
unjustified BOB0

Show that
 $\frac{\ln 4}{4} = \frac{2 \ln 2}{4}$
and conclusion

Examiner's Comments

Many candidates just stated or implied that

$$\frac{\ln 2}{2} - \frac{\ln 4}{4} = 0$$

, either without proof, or by

using their calculator and decimals. These

candidates scored no marks, because of

the "detailed reasoning" instruction.

$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - 1 \times \ln x}{x^2}$	or	$\frac{1}{x} \times \frac{1}{x} + \ln x \times \left(-\frac{1}{x^2}\right)$	oe
---	----	---	----

$\frac{1}{x^2} - \frac{\ln x}{x^2} = 0$	or	$\frac{1 - \ln x}{x^2} = 0$
---	----	-----------------------------

M1
(AO 3.1a)

M1
(AO 1.1)

A1
(AO 1.1)

A1
(AO 1.1)

[4]

Attempt diff, ≥ one
term correct

oe, their $\frac{dy}{dx} = 0$

Allow (e, 0.368) or (e,
0.37)

or (2.7, 0.37) (2 sf)

c

1 - ln x = 0 oe

x = e or 2.72 or 2.7 (2 sf)

Coordinates are (e, $\frac{1}{e}$)

Examiner's Comments

This question was well answered. A few candidates did not find the y -coordinate.
Some made mistakes in the differentiation.

Attempt	$\frac{d^2y}{dx^2}$
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M1
(AO 2.1)

$= \frac{x^2(-\frac{1}{x}) - 2x(1 - \ln x)}{x^4}$	or	$\frac{-3 + 2 \ln x}{x^3}$	oe
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A1
(AO 1.2)

Substitute $x = e$ (or 2.72) into	$\frac{d^2y}{dx^2}$
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M1
(AO 1.1)

$\frac{d^2y}{dx^2} = -\frac{1}{e^3}$	oe or -0.0498
--------------------------------------	---------------

A1
(AO 1.1)

$\frac{d^2y}{dx^2} < 0$, hence maximum
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B1f
(AO 3.2a)

[5]

Attempt diff their
 $\frac{dy}{dx}$
 \frac{dx}

All correct, not necessarily simplified
cao

Sub their x from (c) into
 $\frac{d^2y}{dx^2}$
their

cao Allow or
- 0.0497 or -0.05

ft their result of sub
their x into their
$\frac{d^2y}{dx^2}$ dep see
result

Example of grad method

Sub 2.7 & 2.8 in
$\frac{dy}{dx}$ M1

0.00093, -0.0038
A1A1

State grad +ve & -ve or show on diag dep
A1A1 M1

Hence max B1f dep
M1A1A1

No proof, no marks

				<p><u>Examiner's Comments</u></p> <p>Most candidates attempted a correct method. Some made mistakes in the differentiation. Others made numerical errors when substituting $x = e$ into $\frac{d^2y}{dx^2}$</p> <p>Some considered the gradient on either side of the turning point, generally correctly. In both this part and part (c), candidates who used "e" throughout, rather than its approximate decimal value, produced neater and more efficient solutions.</p>	
		Total	13		
14	a	$f'(x) = -12x(x^2 + a)^{-2}$ for $x > 0$, $-12x < 0$ and $(x^2 + a)^2 > 0$ negative divided by positive is always negative, hence function is decreasing	M1 (AO 3.1a) A1(AO 2.1) M1 (AO 2.1) E1(AO 2.4) [4]	Attempt differentiation to obtain $kx(x^2 + a)^{-2}$ Obtain fully correct derivative Attempt to show that $f'(x) < 0$ Fully convincing argument	
	b	$f''(x) = -12(x^2 + a)^{-2} + 48x^2(x^2 + a)^{-3}$ $f''(x) = 0$ (and $f'(x) \neq 0$ since $f'(x) = 0$ only when $x = 0$)	M1 (AO 3.1a) A1 (AO 1.1) B1 (AO 1.2)	Attempt use of product, or quotient, rule Obtain correct expression Identify condition for a	Allow unsimplified

		$-12(x^2 + a) + 48x^2 = 0$ $36x^2 = 12a$ $x^2 = \frac{a}{3}$ $x = \sqrt{\frac{a}{3}}, y = \frac{9}{2a}$	<p>M1 (AO 3.1a)</p> <p>A1 (AO 1.1) [5]</p>	<p>point of inflection.</p> <p>Attempt correct process to solve for x</p> <p>Obtain correct coordinates</p>	<p>Seen or implied</p> <p>Accept non-stationary condition omitted</p> <table border="1" data-bbox="1456 502 1825 590"> <tr> <td data-bbox="1456 502 1568 590">AO if</td> <td data-bbox="1568 502 1825 590">$x = \pm\sqrt{\frac{a}{3}}$</td> </tr> </table>	AO if	$x = \pm\sqrt{\frac{a}{3}}$	
AO if	$x = \pm\sqrt{\frac{a}{3}}$							
		Total	9					