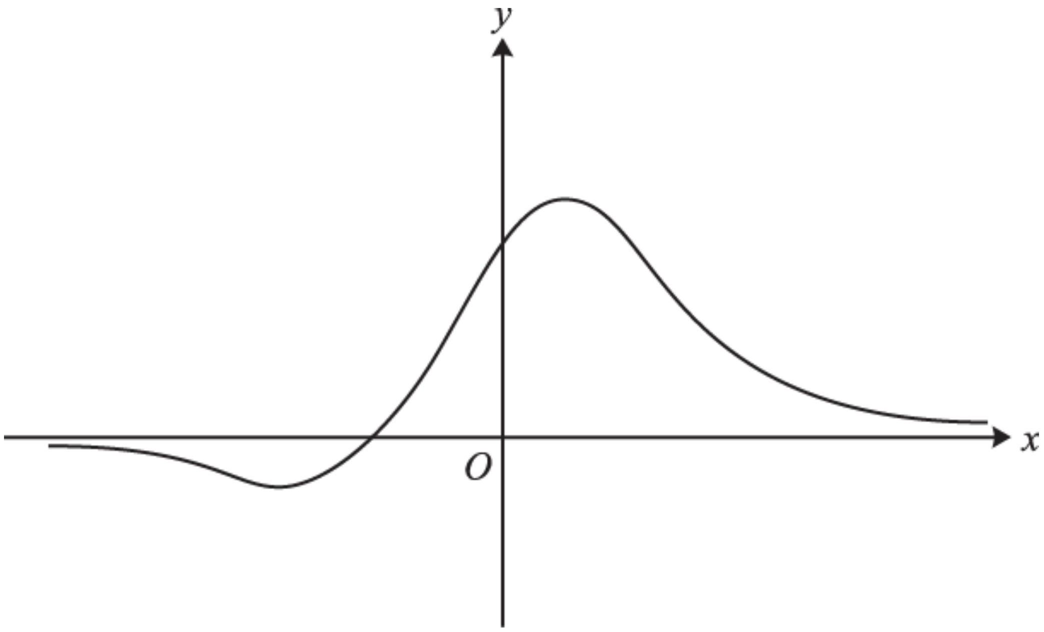
1.



The diagram shows the curve $y = \dfrac{2x+4}{x^2+5}$.

i. Find $\dfrac{dy}{dx}$ and hence find the coordinates of the two stationary points.

[6]

ii. The function g is defined for all real values of x by

$$g(x) = \left|\frac{2x+4}{x^2+5}\right|.$$

a. Sketch the curve $y = g(x)$ and state the range of g.

[3]

b. It is given that the equation $g(x) = k$, where k is a constant, has exactly two distinct real roots.
Write down the set of possible values of k.

[2]

2. Find the equation of the tangent to the curve $y = \dfrac{5x+4}{3x-8}$ at the point , (2, −7).

[5]

3. A curve has equation $x = (y + 5)\ln(2y - 7)$.

(a) Find $\frac{dx}{dy}$ in terms of y. [3]

(b) Find the gradient of the curve where it crosses the y-axis. [5]

4. A sheet of metal is a square of side 21 cm. Equal squares of side x cm are cut from each corner, and the sheet is then folded to make an open box with vertical sides.

(a) Use calculus to find the value of x that maximises the volume of the box. Justify that the volume is a maximum. [6]

(b) State an assumption that is needed when answering part **(a)**. [1]

5. **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{\ln x}{x}$.

(a) Find the x-coordinate of the point where the curve crosses the x axis. [2]

(b) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis.

[2]

(c) Find the coordinates of the turning point on the curve. [4]

(d) Determine whether this turning point is a maximum or a minimum. [5]

6. **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{2x}{3x-1} + \sqrt{5x+1}$. Show that the equation of the tangent to the curve at the point where $x = 3$ is $19x - 32y + 95 = 0$. [7]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Attempt use of quotient rule or equiv	M1	condone one slip only but must be subtraction in numerator; condone absence of necessary brackets; or equiv	
		i	Obtain $\dfrac{2(x^2+5)-2x(2x+4)}{(x^2+5)^2}$	A1	or correct equiv; now with brackets as necessary	correct numerator but error in denominator: max M1A0A1M1A1A1; numerator wrong way round:
		i	Obtain $-2x^2 - 8x + 10 = 0$	A1	or equiv involving three terms	max M0A0A0M1A1A1
		i	Attempt solution of three-term quadratic equation based on numerator of derivative (even if their equation has no real roots)	M1	implied by no working but 2 correct values obtained	M1 for factorisation awarded if attempt is such that x^2 term and one other term correct upon expansion; if formula used, M1 awarded as per Qn 2
		i	Obtain -5 and 1	A1		
		i	Obtain $(-5, -\frac{1}{5})$ and (1, 1)	A1	Allow $-\dfrac{6}{30}$	
		ii	Sketch (more or less) correct curve	B1	showing negative part reflected in x-axis and positive part unchanged; ignore intercept values on axes, right or wrong	
		ii	State values between 0 and their y-value of maximum point lying in first quadrant	M1	accept ≤ or < signs here	
		ii	State correct $0 \le y \le 1$	A1ft	following their y-value of maximum point in first quadrant; now with ≤ signs; or equiv perhaps involving g or g(x)	for "$y \ge 0$ and $y \le 1$", award M1A1; for separate statements $y \ge 0$, $y \le 1$, award M1A0

		ii	Indicate, in some way, values between y-coordinates of maximum point and reflected minimum point (provided their y-coordinate of minimum point is negative)	M1	allow ≤ sign(s) here; could be clear indication on graph	for "$k > \frac{1}{5}$ and $k < 1$" and, award M1A1; for separate statements, award M1A0
		ii	State $\frac{1}{5} < k < 1$	A1	or correct equiv; not ≤ now; correct answer only earns M1A1 **Examiner's Comments** Part (i) of this question involved a routine process and most candidates proceeded accordingly using the quotient rule. There were some errors including omitting the square in the denominator and getting the two parts of the numerator the wrong way round. Lack of precision with algebra led to further errors among which the commonest was the simplification of $2(x^2 + 5) - 2x(2x + 4)$ to give $2x^2 + 10 - 4x^2 + 8x$. This particular mistake led to both stationary points having positive y-coordinates. Candidates making this elementary algebraic mistake seemed totally unconcerned by the fact that the coordinates of their stationary points did not match the evidence of the curve shown in the question. The requests in part (ii) were much more challenging and many candidates lacked the necessary skill and insight to answer them successfully. Most drew an acceptable curve of $y = g(x)$ although not all gave sufficient attention to detail, particularly at either end of the x-axis. Perhaps mindful of the fact that modulus was involved, many candidates opted for $g(x) \geq 0$ for the range of g; they did not seem to use the evidence provided by their graph. Only 33% of candidates earned three marks forpart (ii)(a). Fewer candidates succeeded with the final part; those that did were usually able to write down the correct answer by consideration of their graph from the previous part. But most candidates were unable to make a sensible attempt or the request for two distinct real roots sent them into a $b^2 - 4ac > 0$ routine.	
			Total	11		
2			Attempt use of quotient rule or, after adjustment, product rule	*M1	For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x - 8$ in denominator; allow M1 for numerator the wrong way round	For product rule attempt, *M1 for $k_1(3x - 8)^{-1} + k_2(5x + 4)(3x - 8)^{-2}$ form and A1 for correct constants 5 and −3;

			Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv	A1	Allow if missing brackets implied by subsequent simplification or calculation	
			Substitute 2 to obtain −13 or equiv	A1		
			Attempt to find equation of tangent	M1	Dep *M; equation of tangent not normal	
			Obtain $y = -13x + 19$ or $13x + y - 19 = 0$	A1	Or similarly simplified equiv with 3 non-zero terms **Examiner's Comments** This opening question was answered very well in general with 74% of the candidates recording full marks. The majority applied the quotient rule accurately although lack of care with brackets in the numerator did lead to some sign errors. Some candidates opted for use of the product rule and this was not handled quite so convincingly. There were some cases where candidates stopped as soon as they had found the gradient but, in general, candidates proceeded without difficulty to produce the equation of the tangent and to present it in an acceptable form.	
			Total	5		
3		a	Attempt use of product rule Obtain ln $(2y-7)$ Obtain.... $+\frac{2(y+5)}{2y-7}$	M1(AO1.1a) A1(AO1.1) A1(AO1.1) [3]	Award for sight of two terms	
		b	$(y+5)\ln(2y-7) = 0$ $y = -5$ or $y = 4$	M1(AO1.1) M1(AO3.1a)	Substitute $x = 0$ and attempt to solve May attempt to form $\frac{dy}{dx}$ by attempting to form	

			Substitute $y = 4$ into $\dfrac{dx}{dy}(= \ln 1 + 18)$ Obtain $\dfrac{dy}{dx} = \dfrac{1}{18}$ Substitute $y = -5$ into $\dfrac{dx}{dy}$ (or x) and indicate that ln (–17) does not exist	A1(AO1.1) M1(AO2.1) A1(AO2.3) [5]	the reciprocal. Allow any attempt however poor Do not allow ln |–17| May state that the ln graph does not exist for negative values or at (0, –17)	
			Total	8		
4		a	$V = x(21 - 2x)^2$ $= 4x^3 - 84x^2 + 441x$ $V' = 12x^2 - 168x + 441$ $12x^2 - 168x + 441 = 0$ $x = 3.5$ cm when $x = 3.5$, $V'' = 24 \times 3.5 - 60 < 0$ hence maximum	B1(AO3.3) M1(AO1.1a) M1(AO3.1b) A1ft(AO3.2a) M1(AO1.1) A1ft(AO2.1) [6]	Sate correct expression for volume — oe Expand and attempt differentiation — Or use product rule Equate to 0 and attempt to solve — **BC** Obtain $x = 3.5$ cm only, ft on their V — A0 if 10.5 also given Use second derivative oe Conclude maximum — If evaluated, must be correct	

					Evidence required	
		b	Accept any sensible assumption	B1(AO3.5b) [1]	E.g. Thickness of metal is assumed negligible	
			Total	7		
5		a	DR $\frac{\ln x}{x} = 0$ $\Rightarrow \ln x = 0$ or $\frac{\ln 1}{1} = 0$ $\Rightarrow x = 1$	M1 (AO 1.1a) A1 (AO 1.1) [2]	May not be seen May be implied **Examiner's Comments** This question was well answered.	
		b	y-coordinates are $\frac{\ln 2}{2}$ and $\frac{\ln 4}{4}$ $\frac{\ln 4}{4} = \frac{2\ln 2}{4} = \frac{\ln 2}{2}$ oe	B1* (AO 1.1) B1dep* (AO 3.1a)	Allow $\frac{\ln 4}{4} = \ln 4^{\frac{1}{4}} = \ln\sqrt{2} = \frac{\ln 2}{2}$ Both = 0.346... B0B0	

			⇒ *AB* is / to *x*-axis **AG**	[2]	<table><tr><td>Show that $\frac{\ln 4}{4} = \frac{2\ln 2}{4}$ and conclusion</td><td>use of $\frac{\ln 4}{4} - \frac{\ln 2}{2} = 0$ unjustified B0B0</td></tr></table> Examiner's Comments Many candidates just stated or implied that $\frac{\ln 2}{2} - \frac{\ln 4}{4} = 0$, either without proof, or by using their calculator and decimals. These candidates scored no marks, because of the "**detailed reasoning**" instruction.	
		c	<table><tr><td>$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - 1 \times \ln x}{x^2}$</td><td>or</td><td>$\frac{1}{x} \times \frac{1}{x} + \ln x \times (-\frac{1}{x^2})$</td><td>oe</td></tr><tr><td>$\frac{1}{x^2} - \frac{\ln x}{x^2} = 0$</td><td>or</td><td>$\frac{1 - \ln x}{x^2} = 0$</td><td></td></tr></table> 1 − ln x = 0 oe x = e or 2.72 or 2.7 (2 sf) Coordinates are (e, $\frac{1}{e}$)	M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 1.1) A1 (AO 1.1) [4]	<table><tr><td>Attempt diff, ≥ one term correct oe, their $\frac{dy}{dx} = 0$ Allow (e, 0.368) or (e, 0.37)</td><td>or (2.7, 0.37) (2 sf)</td></tr></table> Examiner's Comments This question was well answered. A few candidates did not find the *y*-coordinate. Some made mistakes in the differentiation.	

		d	Attempt $\frac{d^2y}{dx^2}$ $= \frac{x^2(-\frac{1}{x})-2x(1-\ln x)}{x^4}$ or $\frac{-3+2\ln x}{x^3}$ oe Substitute x = e (or 2.72) into $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = -\frac{1}{e^3}$ oe or –0.0498 $\frac{d^2y}{dx^2} < 0$, hence maximum	M1 (AO 2.1) A1 (AO 1.2) M1 (AO 1.1) A1 (AO 1.1) B1f (AO 3.2a) [5]	Attempt diff their $\frac{dy}{dx}$ All correct, not necessarily simplified cao Sub their x from (c) into their $\frac{d^2y}{dx^2}$ cao Allow or – 0.0497 or –0.05 ft their result of sub their x into their $\frac{d^2y}{dx^2}$ dep see result Example of grad method Sub 2.7 & 2.8 in $\frac{dy}{dx}$ M1 0.00093, –0.0038 A1A1 State grad +ve & –ve or show on diag dep A1A1 M1 Hence max B1f dep M1A1A1 No proof, no marks Examiner's Comments Most candidates attempted a correct method. Some made mistakes in the	

					differentiation. Others made numerical errors when substituting $x = \mathrm{e}$ into $\frac{d^2y}{dx^2}$ Some considered the gradient on either side of the turning point, generally correctly. In both this part and part (c), candidates who used "e" throughout, rather than its approximate decimal value, produced neater and more efficient solutions.	
			Total	13		
6			DR $\frac{2(3x-1)-6x}{(3x-1)^2}$ $\frac{5}{2}(5x+1)^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{-2}{(3x-1)^2} + \frac{5}{2}(5x+1)^{-\frac{1}{2}}$ at $x = 3$, $\frac{dy}{dx} = -\frac{2}{64} + \frac{5}{8} = \frac{19}{32}$	M1 (AO 1.1) M1 (AO 1.1) A1 (AO 2.1) M1 (AO 1.1a) B1 (AO 1.1)	Attempt to differentiate first term using quotient rule or equiv Attempt to differentiate second term — Obtain $k(5x+1)^{-\frac{1}{2}}$ Fully correct derivative — Allow unsimplified Attempt gradient at $x = 3$	

			at $x = 3$, $y = \frac{19}{4}$ $y - \frac{19}{4} = \frac{19}{32}(x-3)$ $32y - 152 = 19x - 57$ $19x - 32y + 95 = 0$ AG	M1 (AO 2.1) A1 (AO 2.1) [7]	Correct y-coordinate Attempt equation of line with their y-coordinate and gradient Rearrange to given form	At least one line of working seen
			Total	7		