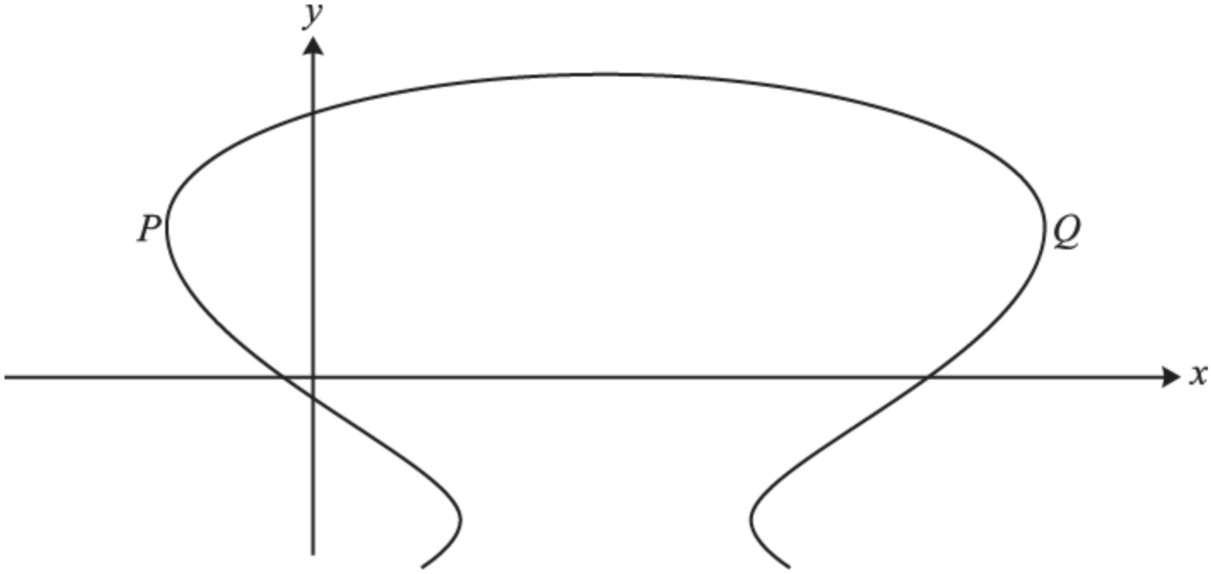
1. The equation of a curve is $xy^2 = x^2 + 1$. Find $\frac{dy}{dx}$ in terms of x and y, and hence find the coordinates of the stationary points on the curve.

[7]

2.



The diagram shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q.

[8]

3. A curve has equation $(x + y)^2 = xy^2$. Find the gradient of the curve at the point where $x = 1$.

[7]

4. Given that $y\sin 2x + \frac{1}{x} + y^2 = 5$, find an expression for $\frac{dy}{dx}$ in terms of x and y.

[5]

5. **In this question you must show detailed reasoning.**

Find the exact values of the x-coordinates of the stationary points of the curve $x^3 + y^3 = 3xy + 35$.

[9]

6. **In this question you must show detailed reasoning.**

A curve has equation

$$x \sin y + \cos 2y = \frac{5}{2}$$

for $x \geq 0$ and $0 \leq y < 2\pi$.

Determine the exact coordinates of each point on the curve at which the tangent to the curve is parallel to the y-axis. [9]

7. The equation of a curve is $4\sqrt{y} + x^2 y - 8 = 0$. The curve meets the line $y = 1$ at two points. Find the gradient [7]

of the curve at each of these points.

8. **In this question you must show detailed reasoning.**

Show that the curve with equation $x^2 - 4xy + 8y^3 - 4 = 0$ has exactly one stationary point. [10]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			For attempt at product rule on xy^2	M1	or changing equation to $y^2 = x + x^{-1}$	
			$\frac{d}{dx}\left(y^2\right) = 2y\frac{dy}{dx}$	B1	soi in the differentiating process	
			$\frac{dy}{dx} = \frac{2x - y^2}{2xy}$ or $\frac{1 - x^{-2}}{2y}$	A1	Award B1 for $(\pm)\frac{1}{2}\left(x + x^{-1}\right)^{-½}(1$	
			Stationary point $\rightarrow$ (their) $\frac{dy}{dx} = 0$ soi	M1		
			$x^2 = 1$ or $y^2 = 2$ or $y^4 = 4$	A1	Ignore any other values Accept 1.41 or $4^{¼}$ for $\sqrt{2}$ **Examiner's Comments**	
			$\left(1, \sqrt{2}\right), \quad \left(1, -\sqrt{2}\right)$	A1,A1	The first part was generally answered well and most obtained the correct expression for though a few equated to 0 at an earlier stage (so losing a simple mark). The derivation of $x^2 = 1$ or $y^4 = 4$ was well done but the final easy hurdle of obtaining the two (and only two) pairs of coordinates left much to be desired.	SR. Award A1 only if extra co-ordinates presented with both correct answers
			Total	7		
2			$3y^2\frac{dy}{dx}$	B1	or $2x\frac{dx}{dy}$	if B0B0 M0

			$2x-12\frac{dy}{dx}-8$	B1	$3y^2-8\frac{dx}{dy}-12$	SC2 for $\frac{dy}{dx}=$
			their $3y^2\frac{dy}{dx}-12\frac{dy}{dx}=8-2x$ soi	M1	their $2x\frac{dx}{dy}-8\frac{dx}{dy}=-3y^2+12$	$\frac{1}{3}(-x^2+8x+12y+4)^{\frac{-2}{3}}\times(-2x$
					must be two terms on each side must follow from RHS = 0	
					This mark may be implied if	
			must be two terms on each side and must follow from RHS = 0			
			$\frac{dy}{dx}=\frac{8-2x}{3y^2-12}$ oe	A1	$\frac{dx}{dy}=0$	M1 may be earned for setting correct denominator equal to 0
					is substituted and there is no evidence for an incorrect expression for $\frac{dx}{dy}$	
			their $3y^2-12=0$	M1*		$x\neq 4$ not required
			$y=(\pm)\,2$	A1	A0 if $\frac{dy}{dx}$ incorrect	
			substitution of their positive y value in original equation	M1dep*		ignore substitution of – 2
					A0 if $\frac{dy}{dx}$ incorrect	
			$x=10$, $x=-2$ and no others cao	A1	<u>Examiner's Comments</u> Very many candidates showed mastery of implicit differentiation, and an overwhelming majority achieved the first 4 marks on this question. Many went on successfully to score full marks. However, weaker candidates set	condone omission of formal statement of coordinates (10, 2) and (–2, 2)

					$\frac{dy}{dx}$ equal to zero and made no further progress. Surprisingly, solving $3y^2 - 12 = 0$ often led to $y = \pm 4$.	
			Total	8		
3			LHS is $k(x+y)(1+\frac{dy}{dx})$	M1	or $2x + 2y\frac{dy}{dx} + ky + kx\frac{dy}{dx}$ *k* is any positive integer	some terms may appear on RHS with signs reversed
			$k = 2$	A1		if **M0** in middle scheme, **SC1** for **three terms out of four** completely correct with $k = 2$
			$2y\frac{dy}{dx}$ on RHS from differentiating y^2	B1		may appear on LHS with sign reversed
			$y^2 + Kxy\frac{dy}{dx}$ on RHS	M1	*K* is any positive integer	NB $K = 2$; may appear on LHS with signs reversed
			obtains a value of *y* from eg $(1 + y)^2 = 1 \times y^2$ oe	M1	allow even if follows incorrect manipulation	NB $y = -0.5$
			substitution of $x = 1$ and their *y* dependent on at least two correct terms seen following differentiation, even if follows subsequent incorrect manipulation	M1	may be implied by $1+\frac{dy}{dx} = \frac{1}{4} - \frac{dy}{dx}$	or $\frac{dy}{dx} = \frac{2-1-0.25}{-1-2+1}$ NB $\frac{dy}{dx} = \frac{2x+2y-y^2}{2xy-2x-2y}$
			$\frac{dy}{dx} = -\frac{3}{8}$ oe cao	A1		-0.375

					Examiner's Comments Very many candidates showed mastery of implicit differentiation, and an overwhelming majority earned the first 4 marks on this question. Many went on successfully to score full marks. However, some weaker candidates set $\frac{dy}{dx}$ equal to zero and made no further progress, or lost the accuracy mark either because their value of y was incorrect or because their attempt to make $\frac{dy}{dx}$ the subject of the formula went astray. A small number of candidates attempted to make y the subject of the equation before differentiating. This was nearly always unsuccessful as the crucial branch of the curve was usually ignored.	
			Total	7		
4			$2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	from differentiation of y^2	
			$\sin 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x$	M1	correct use of Product Rule	allow sign error or one incorrect coefficient
			$\sin 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x-\frac{1}{x^2}+2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1		
			$(\sin 2x+2y)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x^2}-2y\cos 2x$ oe	M1	collection of like terms on separate sides, need not be factorised	must be two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$
			$[\frac{\mathrm{d}y}{\mathrm{d}x}=]\frac{1-2x^2y\cos 2x}{(\sin 2x+2y)x^2}$ oe isw	A1	eg $[\frac{\mathrm{d}y}{\mathrm{d}x}=]\frac{x^{-2}-2y\cos 2x}{(\sin 2x+2y)}$	A0 for eg *y*… **Examiner's Comments**

						This question was done very well indeed, with many candidates achieving full marks. A common error was to differentiate the second term as lnx and some candidates made sign or coefficient errors when using the product rule.
			Total	5		
5			DR $3x^2 + 3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$ $= 3y + 3x\frac{\mathrm{d}}{\mathrm{d}}$ To find the stationary points let $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $y = x^2$ $x^3 + (x^2)^3 = 3x(x^2) + 35$ $x^6 - 2x^3 - 35 = 0$ Let $p = x^3$, then $p^2 - 2p - 35 = 0$ $p = 7$ or -5 $\Rightarrow x = \sqrt[3]{7}$ or $x = -\sqrt[3]{5}$	B1(AO1.1) M1(AO3.1a) A1(AO1.1) E1(AO2.1) M1(AO3.1a) M1(AO2.1) M1(AO2.1) M1(AO1.1) A1(AO3.2a)	Attempt LHS derivative | Two non-constant terms Attempt product rule on RHS Correct on RHS Explicitly set their derivative equal to zero | Alternate $x = y^{\frac{1}{2}}$ Attempt to solve for their y or their x Substitute to get their | Alternate $y^3 - 2y^{\frac{3}{2}} - 35 = 0$	

				[9]	polynomial in one variable Transform their disguised quadratic Solve their 3 term quadratic For both correct	A0 for decimal answer
			Total	9		
6			DR $\sin y + x\cos y\frac{dy}{dx} - 2\sin 2y\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{\sin y}{2\sin 2y - x\cos y}$ $2\sin 2y - x\cos y = 0$	B1(AO1.1a) M1(AO1.1) A1(AO1.1) M1(AO3.1a) M1(AO3.1a)	Correct derivatives of cos$y$ and – 2sin2y Attempt use of product rule for $x$sin$y$ Obtain correct derivative	

			Answer	Marks	Guidance	
			$4\sin y\cos y - x\cos y = 0$ $\cos y(4\sin y - x) = 0$ so $\cos y = 0$ or $x = 4\sin y$ $\cos y = 0$ gives $(\frac{7}{2}, \frac{1}{2}\pi)$ $x = 4\sin y$ gives $4\sin^2 y + \cos 2y = 2.5$ $4\sin^2 y + 1 - 2\sin^2 y = 2.5$ $\sin y = \pm\frac{1}{2}\sqrt{3}$ $\sin y = \frac{1}{2}\sqrt{3}$ gives $(2\sqrt{3}, \frac{1}{3}\pi)$ and $(2\sqrt{3}, \frac{2}{3}\pi)$ $\sin y = -\frac{1}{2}\sqrt{3}$ gives $x < 0$, so no valid solutions	A1(AO2.1) M1(AO3.1a) A1(AO3.2a) A1(AO2.4) [9]	Rearrange and use denominator = 0 Use $\sin 2y = 2\sin y\cos y$ and attempt solution Obtain $(\frac{7}{2}, \frac{1}{2}\pi)$ Substitute $x = 4\sin y$ into original equation and attempt to solve Obtain one correct solution Obtain both correct roots	Including use of correct identity Must discount $\sin y = -\frac{1}{2}\sqrt{3}$
			Total	9		
7			$Ay^{-\frac{1}{2}} \times \frac{dy}{dx}$ $Bxy + x^2\frac{dy}{dx}$	M1 M1	A is a constant B is a constant	

			Answer	Marks	Guidance	
			$4\times\frac{1}{2}y^{-\frac{1}{2}}\times\frac{dy}{dx}+2xy+x^2\frac{dy}{dx}[=0]$	A1		NB $\frac{-2xy}{x^2+2y^{-\frac{1}{2}}}$
			$x = \pm 2$	B1	both values	
			substitution of (*their* 2, 1) *or* (*their* –2, 1) following differentiation	M1	may follow incorrect rearrangement	from $4\sqrt{1}+x^2\times1-8=$
			at (2, 1) $m = -\frac{2}{3}$	A1		
			at (– 2, 1) $m = \frac{2}{3}$	A1		
			Alternatively, marks for differentiation may be awarded as follows	[7]		association between point and gradient may be evidenced by substitution
			$2x\frac{dx}{dy}$	B1		
			$2x\frac{dx}{dy}y+x^2\times1$	M1	use of Product Rule	
			$2x\frac{dx}{dy}y+x^2+2y^{-\frac{1}{2}}[=0]$	A1		

Examiner's Comments

As in previous years, this topic is well understood and there were many very good responses to this question. A few slipped up in finding the values of x, but most differentiated successfully and then rearranged to make $\frac{dy}{dx}$ the subject

					of their equation. A significant minority made a sign error at this point, thus losing the accuracy,marks at the end following substitution of usually correct x values.	
			Total	7		
8			DR $2x - 4y - 4x\frac{dy}{dx} + 24y^2\frac{dy}{dx} = 0$ $\frac{dy}{dx} = 0$ $2x - 4y = 0$ $x^2 - 2x^2 - x^3 + 4 = 0$ $x^3 - x^2 - 4 = 0$ f(2) = 0 $(x - 2)(x^2 + x + 2) = 0$	M1*(AO1.1a) A1(AO1.1) M1d*(AO1.1a) A1(AO1.1) M1(AO1.1a) A1(AO1.1) B1(AO3.1a) M1(AO2.1) A1(AO2.1) E1(AO2.4) [10]	Attempt implicit differentiation — Deal with at least one y term correctly Obtain correct derivative Either rearrange and use, or substitute Obtain $2x - 4y = 0$, or equiv Eliminate x or y from eqn of curve — **OR** $4y^2 - 8y^2 + 8y^3 - 4 = 0$ Obtain correct cubic — **OR** $2y^3 - y^2 - 1 = 0$ Identify $x = 2$ as root or $(x - 2)$ as factor — **BC** **OR** f(1) = 0 Attempt to — **OR**$(y - 1)(2y^2 + y$	

			Δ = −7 < 0 so quadratic has no real roots, hence just one stationary point		<table><tr><td>factorise cubic - any valid method Correct quadratic quotient Justify one stationary point</td><td>+ 1) = 0 Allow for dividing by root of their cubic Correct working only</td></tr></table>
			Total	10	