1. A curve has equation $x^2 + 2y^2 = 4x$.

   i. By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y.

   [3]

   ii. Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.]

   [3]

2. A curve has implicit equation $y^2 + 2x \ln y = x^2$.

   Verify that the point (1, 1) lies on the curve, and find the gradient of the curve at this point.

   [6]

3. Fig. 6 shows part of the curve $\sin 2y = x - 1$. P is the point with coordinates $(1.5, \frac{1}{12}\pi)$ on the curve.

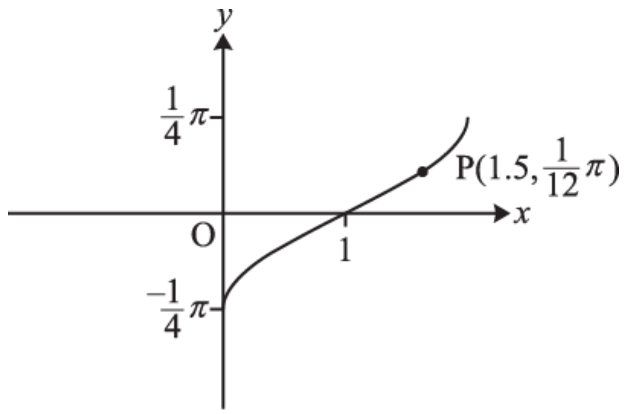


Fig. 6

   i. Find $\frac{dy}{dx}$ in terms of y.

   Hence find the exact gradient of the curve $\sin 2y = x - 1$ at the point P.

   [4]

   The part of the curve shown is the image of the curve $y = \arcsin x$ under a sequence of two geometrical transformations.

   ii. Find y in terms of x for the curve $\sin 2y = x - 1$.

   Hence describe fully the sequence of transformations.

   [4]

4. Fig. 12 shows the curve $2x^3 + y^3 = 5y$.

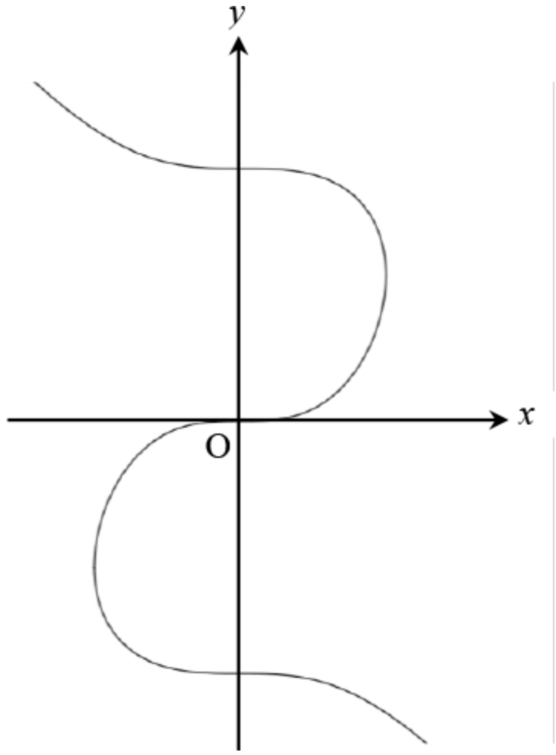


Fig. 12

(a) Find the gradient of the curve $2x^3 + y^3 = 5y$ at the point (1, 2), giving your answer in exact form. [4]

(b) Show that all the stationary points of the curve lie on the y-axis. [2]

5. A curve has equation $3x^{\frac{2}{3}} + 2y^{\frac{1}{3}} = 7$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y. [3]

(ii) Hence find the gradient of the curve at the point with coordinates (1, 8). [2]

6. **You must show detailed reasoning in this question.**

The equation of a curve is

$$y^3 - xy + 4\sqrt{x} = 4 \ .$$

Find the gradient of the curve at each of the points where $y = 1$. [9]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$2x+4y\frac{dy}{dx}=4$	M1	$4y\frac{dy}{dx}$	Rearranging for y and differentiating explicitly is M0
		i		A1	correct equation	Ignore superfluous dy/dx = … unless used subsequently
		i	$\Rightarrow \quad \frac{dy}{dx}=\frac{4-2x}{4y}$	A1	o.e., but mark final answer **Examiner's Comments** This relatively simple implicit differentiation was very well done by almost all candidates.	
		ii	$\frac{dy}{dx}=0 \Rightarrow x=2$	B1dep	dep correct derivative √2, −√2	
		ii	$\Rightarrow 4+2y^2=8 \Rightarrow y^2=2$, $y=\sqrt{2}$ or $-\sqrt{2}$	B1B1	**Examiner's Comments** Most candidates scored two out of three for the point (2, √2), but missed the y = .√2 solution. In a few cases, the denominator was set to zero, giving y = 0.	can isw, penalise inexact answers of ±1.41 or better once only −1 for extra solutions found from using $y=0$
			Total	6		
2			$y^2+2x\ln y=x^2$ $1^2+2\times1\times\ln1=1^2$ so (1, 1) lies on the curve.	B1	clear evidence of verification needed	at least "1 + 0 = 1"

				M1	d/dx (y^2) = 2ydy/dx	Implicit Differentiation must be correct
				M1	d/dx (2x ln y) = 2ln y + 2x/y dy/dx	must be correct
			$2y\frac{dy}{dx}+2\ln y+2x.\frac{1}{y}.\frac{dy}{dx}=2x$	A1cao		condone dy/dx = … unless pursued
			$[\Rightarrow \frac{dy}{dx}=\frac{2x-2\ln y}{2y+2x/y}]$			
			when $x=1, y=1, \frac{dy}{dx}=\frac{2-2\ln 1}{2+2}$	M1	substituting both x = 1 and y = 1 into their dy/dx or their equation in x, y and dy/dx	$2\frac{dy}{dx}+2\ln 1+2\frac{dy}{dx}=2$
			= ½	A1cao	not from wrong working **Examiner's Comments** Implicit differentiation was well understood, although differentiating the '2xlny' term using the product rule defeated some candidates, and there were some algebraic slips in re-arranging to find dy/dx (which virtually all candidates did before substituting x = 1 and y = 1).	
			Total	6		
3		i	2cos2y dy/dx = 1	M1	k cos 2y dy/dx = 1	or dx/dy = k cos2y, $k$cos2ydy = dx
		i	⇒ dy/dx = 1/(2cos2y)	A1		dy/dx = k cos2y is M0
		i	when x = 1½, y = π/12, dy/dx = 1/(2cos(π/6))	M1*	substituting y = π/12 *dep 1st M1	isw from correct exact answer
		i	$= 1/\sqrt{3}$	A1	or $\sqrt{3}/3$	**Examiner's Comments** This question as also very well done, with half the

						Implicit Differentiation candidates scoring full marks. The implicit differentiation was well understood, though there were the usual blemishes from mixing up the derivative and integral formulae for sin 2y. A few candidates re-arranged the equation to get x in terms of y, then found dx/dy, and then the reciprocal dy/dx.
		ii ii ii ii	$2y = \arcsin(x - 1)$ $\Rightarrow y = \frac{1}{2} \arcsin(x - 1)$ translation of 1 unit in positive x-direction [one-way] stretch s.f. ½ in y-direction	M1 A1 B1 B1	 or $\frac{1}{2} \sin^{-1}(x - 1)$ or translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ not ‘shrink’, ‘squash’ etc	allow ‘shift’, but not ‘move’, vector only is B0 transformations can be in either order Examiner's Comments This question as also very well done, with half the candidates scoring full marks. Re-arranging the given implicit equation to give $y = \frac{1}{2} \arcsin(x - 1)$ was well understood, and the transformations were usually accurately described. Note

						that the preferred terms here are ‘translation’ and ‘one-way stretch’.
			Total	8		
4		a	$6x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=5\frac{\mathrm{d}y}{\mathrm{d}x}\left[\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6x^2}{5-3y^2}\right]$ when $x=1$, $y=2$, $6+12\frac{\mathrm{d}y}{\mathrm{d}x}=5\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{6}{7}$	M1(AO1.1a) A1(AO1.1) M1(AO1.1) A1cao(AO2.1) [4]	implicit differentiation correct substituting $x=1$, $y=2$	
		b	$\frac{\mathrm{d}y}{\mathrm{d}x}=0$ so $6x^2=0$ $x=0$ so all stationary points lie on y-axis	B1(AO1.2) E1(AO2.1) [2]	Substitute $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into their differentiated expression Completion of argument	
			Total	6		
5		i	$2x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{2}{3}}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-3x^{-\frac{1}{3}}y^{\frac{2}{3}}$ o.e.	M1 A1 A1	$\frac{\mathrm{d}}{\mathrm{d}x}(y^{\frac{1}{3}})=\frac{1}{3}y^{-\frac{2}{3}}\frac{\mathrm{d}y}{\mathrm{d}x}$ seen correct equation must simplify 2 / (2/3) = 3	

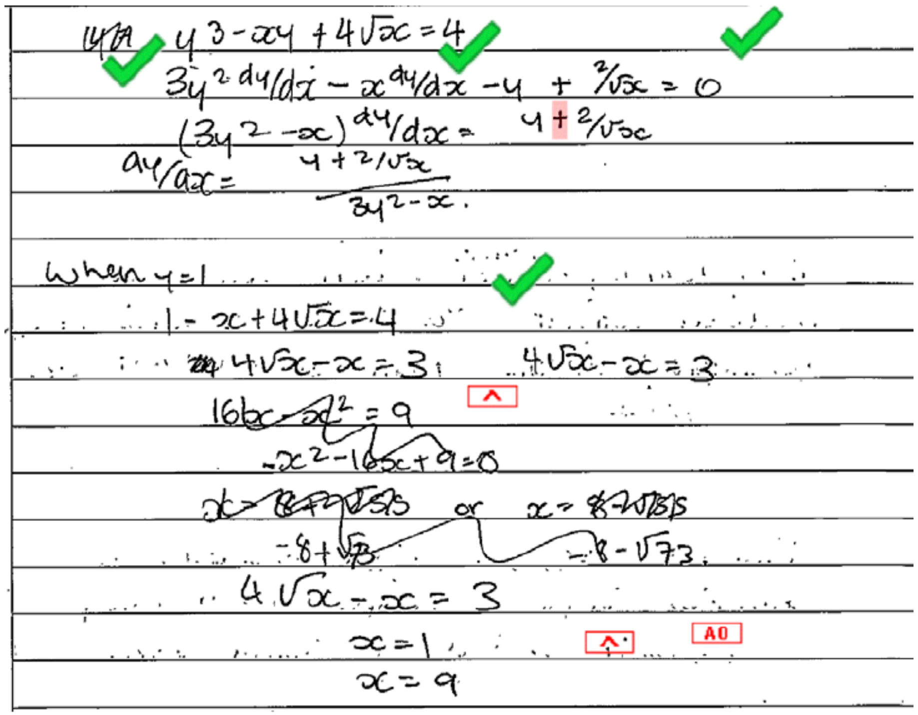
				[3]	<table><tr><td></td><td>mark final answer</td></tr></table> **Examiner's Comments** The implicit derivative here was a straightforward example, and virtually all the candidates got the derivative equation correct. However, simplifying the fractional expression to get the final mark was often missing or incorrect: in particular, many learners made mistakes when dividing 2 by 2/3.	Implicit Differentiation
		ii	when $x = 1$, $y = 8$, $\Rightarrow \frac{dy}{dx} = -3 \times 4 = -12$	M1 A1cao [2]	<table><tr><td>substituting both $x = 1$ and $y = 8$ into their dy/dx NB check power of x is correct in part (i)</td><td></td></tr></table> **Examiner's Comments** There were two easy marks here, and virtually all candidates achieved the 'M' mark for substituting for x and y in their derivative.	
			Total	5		
6			substitution of $y = 1$ $x - 4\sqrt{x} + 3 = 0$ or $4\sqrt{x} = x + 3$ $x = 1$ or 9 $3y^2 \frac{dy}{dx}$	M1 (AO 1.1a) A1 (AO 2.1) A1 (AO 1.1)	<table><tr><td></td><td></td></tr></table>	

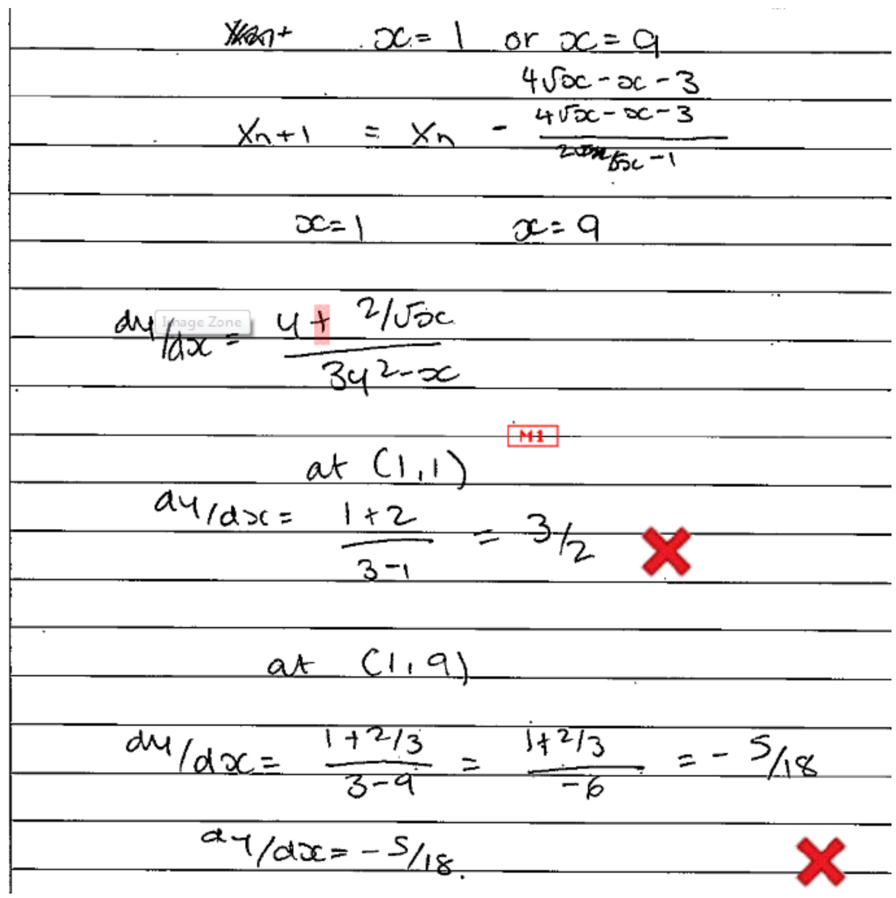
			Answer	Marks	Guidance	
			$-x \times \frac{dy}{dx} - y$ or $x \times \frac{dy}{dx} + y$	B1 (AO 3.1a)		Implicit Differentiation
			$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} - y + \frac{2}{\sqrt{x}}[=0]$	M1 (AO 2.1)	allow one sign error	
			substitution of $y = 1$ and *their* $x = 1$ or *their* $x = 9$			
			$m = -\frac{1}{2}$ [at (1, 1)]	A1 (AO 1.1)		
			$m = -\frac{1}{18}$ [at (1, 9)]	M1 (AO 1.1)	dependent on at least two terms correct on LHS following differentiation	allow following wrong rearrangement after differentiating
				A1 (AO 1.1)	allow – 0.05555…to 2 sf or better	
				A1 (AO 1.1)		
				[9]		

Examiner's Comments

Candidates who did well in this question explained their reasoning clearly, in particular they showed how the values of x were obtained and how $\frac{dy}{dx}$ was obtained.

Candidates who did less well made algebraic and arithmetic slips. They did not fully explain their reasoning, especially

					when finding the values of *x*. Exemplar 3 	Implicit Differentiation

					 This candidate differentiated correctly, but made a sign error when rearranging, which cost accuracy marks at the end. Note that detailed reasoning was required in this question. It was not clear how the (correct) values of x were obtained so A marks were withheld.	Implicit Differentiation
			Total	9		