1.

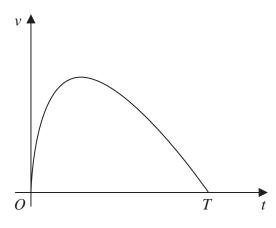


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v \,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t) \ln(t+1)$$
 $0 \le t \le T$

where *t* seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t+1)} - 1 \tag{4}$$

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

- (c) (i) find the value of t_3 to 3 decimal places,
 - (ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

2. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)g(x)$$

where g(x) is a function to be found.

(3)

Given that the curve with equation y = f(x) has at least one stationary point,

(b) find the range of possible values of k.

(3)

3. A curve has equation y = f(x), where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}}$$
 $x > \ln \sqrt[3]{2}$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where *A* and *B* are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4} \tag{2}$$

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation y = x

Using Diagram 1 on the next page,

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of
$$\beta$$

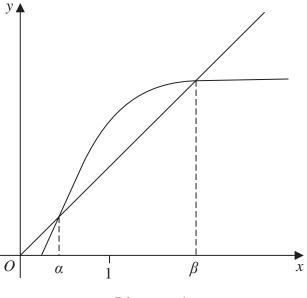
Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

Question 3 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).



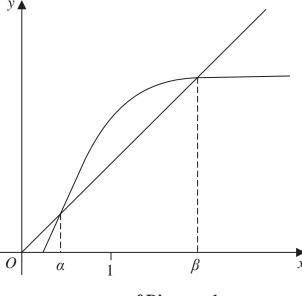


Diagram 1

copy of Diagram 1