1. The curve C has parametric equations

$$x = \sin 2\theta$$
 $y = \csc^3 \theta$ $0 < \theta < \frac{\pi}{2}$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where y = 8

(3)

2.

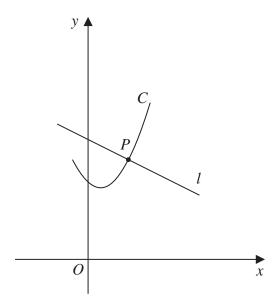


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2\tan t + 1 \qquad \qquad y = 2\sec^2 t + 3 \qquad \qquad -\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{3}$$

The line *l* is the normal to *C* at the point *P* where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2} \tag{5}$$

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2} (x - 1)^2 + 5$$
 (2)

The straight line with equation

$$y = -\frac{1}{2}x + k$$
 where k is a constant

intersects C at two distinct points.

(c) Find the range of possible values for k.

(5)