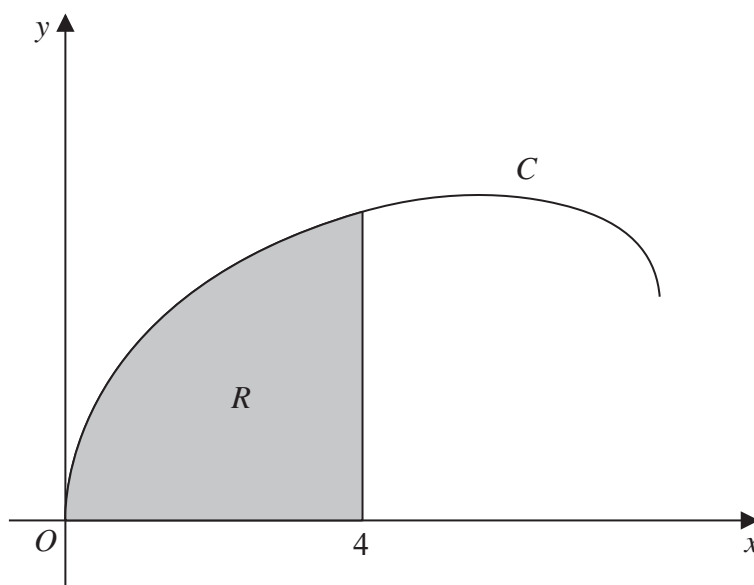


1.



**Figure 6**

Figure 6 shows a sketch of the curve  $C$  with parametric equations

$$x = 8 \sin^2 t \quad y = 2 \sin 2t + 3 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region  $R$ , shown shaded in Figure 6, is bounded by  $C$ , the  $x$ -axis and the line with equation  $x = 4$

(a) Show that the area of  $R$  is given by

$$\int_0^a (8 - 8 \cos 4t + 48 \sin^2 t \cos t) dt$$

where  $a$  is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of  $R$ .

(4)

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2. A curve has equation  $y = f(x)$ , where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where  $A$  and  $B$  are constants to be found.

(5)

(b) Hence show that the  $x$  coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation  $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$  has two positive roots  $\alpha$  and  $\beta$  where  $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for  $\alpha$  and  $\beta$

Diagram 1 shows a plot of part of the curve with equation  $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$  and part of the line with equation  $y = x$

Using Diagram 1 on the next page

(c) draw a staircase diagram to show that the iteration formula starting with  $x_1 = 1$  can be used to find an approximation for  $\beta$

(1)

Use the iteration formula with  $x_1 = 1$ , to find, to 3 decimal places,

(d) (i) the value of  $x_2$

(ii) the value of  $\beta$

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that  $\alpha = 0.432$  to 3 decimal places.

(2)

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3.

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$

(4)

The point  $P(-2, 5)$  lies on the curve.

(b) Find the equation of the normal to the curve at  $P$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers to be found.

(3)

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