Question	Scheme	Marks	AOs
1(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24$ or $\frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\frac{dh}{dt}$	M1	2.1
	$1200\frac{dh}{dt} = 24 - 5h$ *	A1*	1.1b
		(4)	
(b)	$1200\frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24-5h}\,dh = \int dt$ $\Rightarrow$ *e.g.* $\alpha \ln(24-5h) = t(+c)$ oe or $1200\frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24-5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24-5h)$ oe	M1	3.1a
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24-5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24-5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = ...$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $e^{-\frac{t}{240}} > 0$, $h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a

		(2)	
			(12 marks)

Notes

(a)

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ according to the model

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ according to the model

M1: Applies $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V}$ or equivalent correct formula with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ which may be implied by their working

A1*: Correct equation obtained with no errors

Note that: $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48 - 0.1h \Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{0.48 - 0.1h}{24} \Rightarrow 1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h$ * scores B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{\mathrm{d}t}{\mathrm{d}h}$ correctly in terms of h **and** integrates to obtain $t = \alpha \ln(24 - 5h)(+c)$ or equivalent (condone missing brackets around the "24 – 5h") and + c not required for this mark.

A1: Correct equation in any form and + c not required. Do not condone missing brackets unless they are implied by subsequent work.

M1: Substitutes t = 0 and h = 2 to find their constant of integration (there must have been some attempt to integrate)

A1: Correct equation in any form

ddM1: Uses fully correct log work to obtain h in terms of t.

This depends on <u>both</u> previous method marks.

A1: Correct equation

Note that the marks may be earned in a different order e.g.:

$$t + c = -240\ln(24 - 5h) \Rightarrow -\frac{t}{240} + d = \ln(24 - 5h) \Rightarrow Ae^{-\frac{t}{240}} = 24 - 5h$$

$$t = 0, h = 2 \Rightarrow A = 14 \Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = 4.8 - 2.8e^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $Ae^{\alpha t} = 24 - 5h$ (must have a constant "A")

A1: $Ae^{-\frac{t}{240}} = 24 - 5h$

ddM1: Uses t = 0, h = 2 in an expression of the form above to find A

A1: $h = 4.8 - 2.8e^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples

A1: Makes a correct interpretation for their method.
There must be no incorrect working or contradictory statements.
This is not a follow through mark and if their equation in (b) is used it must be correct.

Question	Scheme	Marks	AOs
2 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
(8 marks)			
Notes:			

(a) May be seen in the question so watch out.
B1: Accept 265 thousand or 265 000 or equivalent such as 265 k but not just 265.
(b)
M1: Differentiates to a form $ke^{0.05t}, k > 0, k \neq 220$. Do not be too concerned about the lhs.
M1: Substitutes $t = 10$ into a changed function that was formed from an attempt at differentiation.

The left hand side must have implied differentiation. E.g. Rate = , $N', \frac{dN_b}{dt}, \frac{dN}{dt}$ or even $\frac{dy}{dx}$

A1*: Full and complete proof that requires

- some correct lhs seen at some point. E.g. "Rate = , " $\frac{dN_b}{dt}, \frac{dN}{dt}$ but condone N'.
- an intermediate line/answer of either $11e^{0.05\times10}$ or awrt 18.1 before a minimal conclusion which must be referencing the 18 000 or 18 thousand

(c)

M1: Attempts to set both equations equal to each other and simplify the constant terms.

Look for $220\,e^{0.05t}+35=800\,e^{-0.05t}$ o.e but condone slips

It is also possible to set $\frac{N-45}{220}=\left(e^{0.05t}=\right)\frac{800}{N-10}$ and form an equation in N

A1: Correct quadratic form.

Look for $220\left(e^{0.05t}\right)^2+35e^{0.05t}-800=0$ or $220\,e^{0.1t}+35e^{0.05t}-800=0$ but allow with terms in different order such as $220\,e^{0.1t}+35e^{0.05t}=800$

FYI the equation in N is $N^2-55N-175550=0$

M1: Full attempt to find the value of t (or a constant multiple of t)

This involves the key step of recognising and solving a 3TQ in $e^{0.05t}$ followed by the use of lns.
If the answers to the quadratic just appear (from a calculator) you will need to check.
Accuracy should be to 3sf.
You may see different variables used such as x

$x=e^{0.05t}$, $220\,e^{0.1t}+35e^{0.05t}=800\Rightarrow 220x^2+35x=800\Rightarrow x=1.82...\Rightarrow t=20\ln 1.82...$

Allow use of calculator for solving the quadratic and for $e^{0.05t}=1.82..\Rightarrow t=12.08$

Via the N route it will involve substituting the positive solution to their quadratic into either equation to find a value for t/T using same rules as above.

A1: AWRT 12.08

....

Answers with limited or no working in (b) and (c)

(b) A derivative in the correct form must be seen

(c) Candidates who state $45+220\,e^{0.05t}=10+800\,e^{-0.05t}$ followed by awrt 12.08 (presumably from using num-solv on their calculators) can score SC 1100. Rubric on the front of the paper states that "Answers without working may not gain full credit" so we demand a method in this part.

....

Question	Scheme	Marks	AOs
3(a)	$\frac{dV}{dh} = 200$ oe e.g. $\frac{dh}{dV} = \frac{1}{200}$	B1	1.1b
	$\left(\frac{dh}{dt} =\right) \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}} \Rightarrow \int h^{\frac{1}{2}}\ dh = \int \lambda\, dt \Rightarrow ...h^{\frac{3}{2}} = \lambda t \{+c\}$	M1	1.1b
	$\frac{2}{3}h^{\frac{3}{2}} = \lambda t\{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}} = \lambda t\{+c\}$	A1	1.1b
	$\frac{2}{3}(1.44)^{\frac{3}{2}} = \lambda \times 0 + c \Rightarrow c = 1.152\left(=\frac{144}{125}\right)$	dM1	3.4
	$\frac{2}{3}(3.24)^{\frac{3}{2}} = \lambda \times 8 + "1.152" \Rightarrow \lambda = 0.342\left(=\frac{171}{500}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}} = 0.513t + 1.728$ oe e.g. $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$	A1	3.3
		(5)	
	(b) Alternative:		
	$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}} \Rightarrow \frac{dt}{dh} = \frac{\sqrt{h}}{\lambda} \Rightarrow t = ...h^{\frac{3}{2}}(+c)$	M1	1.1b
	$t = \frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe	A1	1.1b
	$0 = \frac{2(1.44)^{\frac{3}{2}}}{3\lambda} + c$ **and** $8 = \frac{2(3.24)^{\frac{3}{2}}}{3\lambda} + c$ $\Rightarrow \lambda = ...\left(\frac{171}{500}\right)$ **or** $c = ...\left(-\frac{64}{19}\right)$	dM1	3.4
	$\Rightarrow \lambda = ...\left(\frac{171}{500}\right)$ **and** $c = ...\left(-\frac{64}{19}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}} = 0.513t + 1.728$ oe e.g. $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$	A1	3.3
		(5)	
(c)	$5^{\frac{3}{2}} = 0.513t + 1.728 \Rightarrow t = ...$	M1	3.4
	$(t =)$ awrt18.4 min	A1	3.2a
		(2)	
			(10 marks)

Notes

(a)

B1: For $\frac{\mathrm{d}V}{\mathrm{d}h} = 200$ stated or used – may be implied by their chain rule attempt

M1: Requires:

- $\frac{\mathrm{d}V}{\mathrm{d}h} = p, \quad p > 1$
- $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm\frac{k}{\sqrt{h}}$ or e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm\frac{1}{k\sqrt{h}}$ (or a suitable letter for k, which may be λ, but must **not** be a number)
- **application** of the correct chain rule $\left(\frac{\mathrm{d}h}{\mathrm{d}t} = \right)\frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ or any equivalent with $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm\frac{k}{\sqrt{h}}$ or $\pm\frac{1}{k\sqrt{h}}$ and their $\frac{\mathrm{d}V}{\mathrm{d}h}$ correctly placed. So $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{200k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores M0 as $\frac{\mathrm{d}h}{\mathrm{d}V}$ is incorrectly placed.

A1*: A rigorous argument with all steps shown and simplifies to achieve $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\lambda}{\sqrt{h}}$ with no errors.

Do not allow the use of λ for both constants. Allow use of e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm\frac{1}{k\sqrt{h}}$ for full marks.

e.g. $\frac{\mathrm{d}V}{\mathrm{d}h} = 200, \quad \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{200} \times \frac{\lambda}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1M1A0* *unless* e.g. "let $\lambda = \frac{\lambda}{200}$" seen.

Allow correct work leading to e.g. $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{200} \times \frac{k}{\sqrt{h}} \to \frac{\lambda}{\sqrt{h}}$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$ so $\lambda = \frac{k}{200}$

There must be an attempt to link the $\frac{\mathrm{d}h}{\mathrm{d}t}$ with the $\frac{\lambda}{\sqrt{h}}$ which may be missing an = sign.

Allow an argument with $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{k}{\sqrt{h}}$ e.g. $\frac{\mathrm{d}V}{\mathrm{d}h} = 200, \quad \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{200} \times -\frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$

Withhold this A mark if there are notational errors e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = 200, \quad \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{200} \times \frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1(implied)M1A0*

(b) Note that some candidates may work with e.g. $\lambda = \frac{k}{200}$ or e.g. $\lambda = 200k$ which is acceptable.

Candidates who do not have a λ e.g. assume $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{200}{\sqrt{h}}$ then only the first 2 method marks are available (see note below). Condone use of other variables if the intention is clear but the final answer must be in terms of *h* and *t*.

M1: Separates the variables and integrates to obtain an equation of the form $...h^{\frac{3}{2}} = \lambda t\{+c\}$ oe

The constant of integration is not needed for this mark.

A1: $\frac{2}{3}h^{\frac{3}{2}} = \lambda t(+c)$ oe. The constant of integration is not needed for this mark.

Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

dM1: Substitutes $t = 0$ and $h = 1.44$ and attempts to find c.

It is dependent on the previous method mark.

Do not be concerned with the "processing" to find "c" as long as they are using $t = 0$ and $h = 1.44$

May be implied by their value of c.

ddM1: Substitutes $t = 8$ and $h = 3.24$ and their c and attempts to find λ. Do not be concerned with the "processing" to find λ as long as they are using $t = 8$ and $h = 3.24$.

It is dependent on both previous method marks.

A1: Correct equation in the correct form from correct work. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$

Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Note candidates who do not have a λ e.g. assume $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{200}{\sqrt{h}}$ can use either $t = 0$ and $h = 1.44$ **or** $t = 8$ and $h = 3.24$ to find their constant of integration.

(b)Alternative:

M1: Finds the reciprocal of both sides and integrates to obtain an equation of the form $t = ...h^{\frac{3}{2}}(+c)$

A1: $t = \frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe. The constant of integration is not needed for this mark.

dM1: Substitutes $t=0$ and $h=1.44$ **and** substitutes $t=8$ and $h=3.24$ **and** attempts to find λ **or** c.
It is dependent on the previous method mark.
Do not be concerned with the "processing" to find λ or c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$ and reach a value for λ or c. May be implied by their value(s).

ddM1: Complete attempt to find λ **and** c. **It is dependent on both previous method marks.**
Do not be concerned with the "processing" to find λ and c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$.

A1: Correct equation in the correct form. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$
Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

<u>Special Case:</u>

Some candidates are using the given equation in part (b) to find the value of A and the value of B using the given conditions. May score a maximum of 00110. This should be marked as follows:

M0A0: (No attempt to integrate)
M1: Substitutes $t = 0$ and $h = 1.44$ to find a value for B
dM1: Substitutes $t = 8$ and $h = 3.24$ with their value of B to find a value for A
A0: Since they have not used the given model.
(Allow full recovery in (c) if this equation is correct)

(c)

M1: Attempts to substitute $h = 5$ into their equation which must be of the form $h^{\frac{3}{2}} = At + B$ or possibly a rearranged equation e.g. $h^{\frac{1}{2}} = \sqrt[3]{At + B}$ with values of A and B leading to a value for t.
Do not be concerned about the processing as long as they use $h = 5$ and obtain a value for t even if t is negative.

A1: Awrt 18.4 minutes **following a correct equation in (b)**.
The units are required but allow e.g. min, mins, but not just 18.4 and not m (which means metres)
Allow e.g. 18 minutes 25 seconds or 18 mins 26 secs
Note this may follow A0 in part (b) as they may have rearranged incorrectly in (b) but use a correct equation in (c) e.g. $\frac{2}{3}h^{\frac{3}{2}} = \frac{171}{500}t + \frac{144}{125}$, $h^{\frac{1}{2}} = \sqrt[3]{\frac{513}{1000}t + \frac{216}{125}}$ or may come from the special case.
Apply isw following a correct time and units, e.g., 18.4 followed by 18 mins.