Question	Scheme	Marks	AOs
1 (a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{dv}{dt} = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{dv}{dt} = 0 \Rightarrow \frac{(10-0.4\boldsymbol{t})}{(\boldsymbol{t}+1)} = 0.4\ln(t+1)$ **and** then makes progress towards making "$\boldsymbol{t}$" the subject (See notes for this)	dM1	1.1b
	$t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ $t = \frac{26}{1+\ln(t+1)} - 1$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = \frac{26}{1+\ln 8} - 1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
			(8 marks)
Notes:			

(a)
B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)
M1: Attempts to use the product rule in an attempt to differentiate $v = (10-0.4t)\ln(t+1)$

Look for $(10-0.4t) \times \frac{1}{(t+1)} \pm k\ln(t+1)$, where k is a constant, condoning slips.

If you see direct evidence of an incorrect rule used e.g. $vu' - uv'$ it is M0
You will see attempts from $v = 10\ln(t+1) - 0.4t\ln(t+1)$ which can be similarly marked.

In this case look for $\frac{a}{t+1} \pm \frac{bt}{t+1} \pm c\ln(t+1)$

A1: Correct differentiation. Condone a missing left hand or it seen as v', $\frac{dy}{dx}$ or even = 0

$\left(\frac{dv}{dt}\right) = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1}$ or equivalent such as $\left(\frac{dv}{dt}\right) = \frac{10}{t+1} - \frac{0.4t}{(t+1)} - 0.4\ln(t+1)$

dM1: Score for setting their dV/dt = 0 (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring ln(t + 1).
See two examples of how this can be achieved below. It is dependent upon the previous M.
Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow \ln(t+1) = \frac{25-t}{t+1}$$
$$\Rightarrow \ln(t+1) = -1+\frac{26}{t+1}$$

E.g 2

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow 0.4t\ln(t+1)+0.4\ln(t+1) = 10-0.4t$$
$$\Rightarrow 0.4t\left(1+\ln(t+1)\right) = 10-0.4\ln(t+1)$$

A1*: Correctly proceeds to the given answer of $t = \frac{26}{1+\ln(t+1)}-1$ showing all key steps.

The key steps must include

- use of $\frac{\mathrm{d}v}{\mathrm{d}t}$ or v' which must be correct
- a correct line preceding the given answer, usually $t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ or $\frac{26}{t+1}-1 = \ln(t+1)$

(c) (i)
M1: Attempts to use the iteration formula at least once.
Usually to find $t_2 = \frac{26}{1+\ln 8}-1$ which may be implied by awrt 7.44
A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3
(c)(ii)
A1: Uses repeated iteration until value established as awrt 7.33 **seconds**. Allow awrt 7.33 **s**
Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

Question	Scheme	Marks	AOs
2(a)	$\mathrm{f}(x)=\dfrac{\mathrm{e}^{3x}}{4x^2+k}\Rightarrow \mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $\mathrm{f}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}\Rightarrow \mathrm{f}'(x)=3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$	A1	2.1
		(3)	
(b)	If $y=\mathrm{f}(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root	B1	2.2a
	Applies $b^2-4ac\ (\geqslant)\ 0$ with $a=12, b=-8, c=3k$	M1	2.1
	$0<k\leqslant\dfrac{4}{9}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{\alpha\left(4x^2+k\right)\mathrm{e}^{3x}-\beta x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$, $\alpha\beta$>0

condoning bracketing errors/omissions as long as the intention is clear.
If the quotient rule formula is quoted it must be correct.

Condone e.g. $\mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)}$ provided an incorrect formula is not quoted.

May also see product rule applied to $\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}$ to obtain an expression of the form

$\alpha\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}+\beta x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ α,β0<0 condoning bracketing errors/omissions as long as the intention is clear. If the product rule formula is quoted it must be correct.

A1: Correct differentiation in any form with correct bracketing which may be implied by subsequent work.

A1: Obtains $\mathrm{f}'(x)=\left(12x^2-8x+3k\right)\mathrm{g}(x)$ where $\mathrm{g}(x)=\dfrac{\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ **or equivalent**

e.g. $\mathrm{g}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$

Allow recovery from "invisible" brackets earlier and apply isw here once a correct answer is seen.
Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ to $\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ for the final mark.

The "$f'(x) =$" must appear at some point but allow e.g. "$\frac{dy}{dx} =$"

(b) Note that B0M1A1 is not possible in (b)

B1: Deduces that if $y = f(x)$ has at least one stationary point then $12x^2 - 8x + 3k = 0$ has at least one root. There is no requirement to formally state $\frac{e^{3x}}{(4x^2+k)^2} > 0$

This may be implied by an attempt at $b^2 - 4ac \geqslant 0$ or $b^2 - 4ac > 0$ condoning slips.

M1: Attempts $b^2 - 4ac ... 0$ with $a = 12,\ b = -8,\ c = 3k$ where … is e.g. "=", <, >, etc.

Alternatively attempts to complete the square and sets rhs ...0

E.g. $12x^2 - 8x + 3k = 0 \Rightarrow x^2 - \frac{2}{3}x + \frac{1}{4}k = 0 \Rightarrow \left(x - \frac{1}{3}\right)^2 = \frac{1}{9} - \frac{1}{4}k$ leading to $\frac{1}{9} - \frac{1}{4}k \geqslant 0$

A1: $0 < k \leqslant \frac{4}{9}$ but condone $k \leqslant \frac{4}{9}$ and condone $0 \leqslant k \leqslant \frac{4}{9}$

Must be in terms of k not x so do not allow e.g. $0 < x \leqslant \frac{4}{9}$ but condone $\left(0, \frac{4}{9}\right]$ or $\left[0, \frac{4}{9}\right]$

Question	Scheme	Marks	AOs
3(a)	$...xe^x + ...e^x$	M1	1.1b
	$k\left(xe^x + e^x\right)$	A1	1.1b
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b
	$\left(f'(x)=\right)\ \dfrac{(e^{3x}-2)^{\frac{1}{2}}("7"xe^x + "7"e^x) - "\frac{3}{2}"e^{3x}(e^{3x}-2)^{-\frac{1}{2}}\times "7"xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \dfrac{7e^x\left(e^{3x}(2-x)-4x-4\right)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
		(5)	
(b)	$e^{3x}(2-x)-4x-4=0 \Rightarrow x(...e^{3x}\pm...) = ...e^{3x}\pm...$	M1	1.1b
	$\Rightarrow x = \dfrac{2e^{3x}-4}{e^{3x}+4}$ *	A1*	2.1
		(2)	
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
		(1)	
(d)(i)	$x_2 = \dfrac{2e^3-4}{e^3+4} = 1.5017756...$	M1	1.1b
	$x_2 =$ awrt 1.502	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
		(3)	
(e)	$h(x) = \dfrac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297...$ $h(0.4325) = 0.000947...$	M1	3.1a
	Both calculations correct and e.g. states: • There is a change of sign • e.g $f'(x)$ is continuous • $\alpha = 0.432$ (to 3dp)	A1cao	2.4
		(2)	
		(13 marks)	

Notes

(a)

M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $...xe^x \pm ...e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.

A1: $k\left(xe^x + e^x\right)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)

B1: $\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \right) \frac{1}{2}\times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$ (simplified or unsimplified)

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.
Score for achieving an expression of the form

$$(\mathrm{f}'(x)=)\ \frac{(\mathrm{e}^{3x}-2)^{\frac{1}{2}}("7"x\mathrm{e}^{x}+"7"\mathrm{e}^{x})-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\times"7"x\mathrm{e}^{x}}{\mathrm{e}^{3x}-2}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.
Alternatively, applies the product rule. Score for achieving an expression of the form

$$(\mathrm{f}'(x)=)\ (\mathrm{e}^{3x}-2)^{-\frac{1}{2}}("7"x\mathrm{e}^{x}+"7"\mathrm{e}^{x})-"\frac{3}{2}"\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\times"7"x\mathrm{e}^{x}$$ or equivalent (do not be concerned by the constants for their "7" or their "$\frac{3}{2}$" which may be both 1)

Do not condone invisible brackets.

A1: $(\mathrm{f}'(x)=)\ \dfrac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $\mathrm{f}'(x)=$ on the left hand side or allow the use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $$\frac{\mathrm{d}}{\mathrm{d}x}\left(7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)=7\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+7x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$$

$$\Rightarrow\frac{\left(7\mathrm{e}^{x}+7x\mathrm{e}^{x}\right)(\mathrm{e}^{3x}-2)+7x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}=\frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}-2+x\mathrm{e}^{3x}-2x-\frac{3}{2}x\mathrm{e}^{3x}\right)}{(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}\Rightarrow\frac{7\mathrm{e}^{x}\left(\mathrm{e}^{3x}(2-x)-4x-4\right)}{2(\mathrm{e}^{3x}-2)^{\frac{3}{2}}}$$

M1: Attempts the product rule on $x\mathrm{e}^{x}\to\ldots x\mathrm{e}^{x}\pm\ldots\mathrm{e}^{x}$ which may be seen within the expression $\ldots\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\pm\ldots x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+\ldots$ simplified or unsimplified.

A1: $k\left(x\mathrm{e}^{x}+\mathrm{e}^{x}\right)$ which may be seen within the expression $k\left(\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}+x\mathrm{e}^{x}(\mathrm{e}^{3x}-2)^{-\frac{1}{2}}\right)+\ldots$ simplified or unsimplified.

B1: $\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}$ which may be seen within the expression $\ldots+k\left(x\mathrm{e}^{x}\times\left(-\frac{1}{2}\right)\times3\mathrm{e}^{3x}(\mathrm{e}^{3x}-2)^{-\frac{3}{2}}\right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) **Note that if they do not have values $A=-4,\ B=-4$ in (a) (which may be seen later) then maximum score is M1A0***

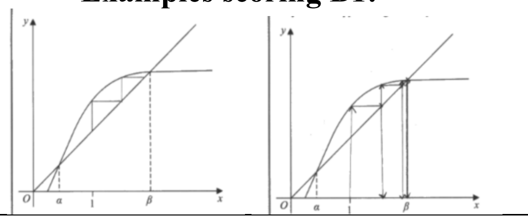
M1: Sets their $\mathrm{e}^{3x}(2-x)-4x-4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of "-4" and "-4"

A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \dfrac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

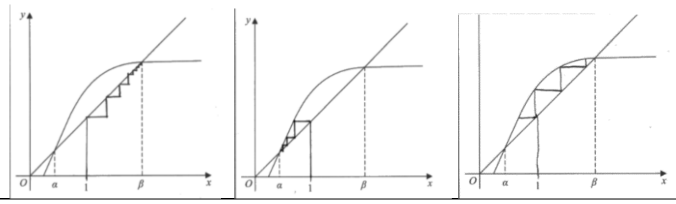
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β. Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the x-axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728…
If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$
You will need to check their calculation is correct.
Other possible functions include:

- $h(x) = x - \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$
- their $f'(x) = \pm\left(\dfrac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x} - 2)^{\frac{3}{2}}}\right)$

  (If correct A and B then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)
- their $g(x) = \pm\left(e^{3x}(2-x) - 4x - 4\right)$

  (If correct A and B then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)

  Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g. $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$", "so rounds to 0.432". Do not allow "hence root"