

1. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a , b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q .

(5)

$$a) \quad px^3 + qxy + 3y^2 = 26$$

$$\frac{d}{dx}(qxy) = qy + qx \frac{dy}{dx} \quad (1)$$

$$3px^2 + qy + qx \frac{dy}{dx} + 6y \frac{dy}{dx} = 0 \quad (1)$$

$$3px^2 + qy + \frac{dy}{dx}(qx + 6y) = 0$$

$$\frac{dy}{dx}(qx + 6y) = -3px^2 - qy \quad (1)$$

$$\frac{dy}{dx} = \frac{-3px^2 - qy}{qx + 6y} \quad (1)$$



b) $P(-1, -4)$ lies on C :

$$p(-1)^3 + q(-1)(-4) + 3(-4)^2 = 26 \quad (1)$$

$$\begin{array}{rcl} -p & +4q & +48 \\ & & = 26 \\ & & -p + 4q = -22 \quad (1) \end{array}$$

Normal to C at P has equation $19x + 26y + 123 = 0$

$$\Rightarrow y = -\frac{19}{26}x - \frac{123}{26} \quad \therefore m = -\frac{19}{26} \quad (1)$$

$$m_{\text{normal}} = \frac{-1}{m_{\text{tangent}}}$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=-1 \\ y=-4}} = -\frac{1}{-\frac{19}{26}} = \frac{26}{19}$$

solve (1) and (2) simultaneously using calculator:

$$\Rightarrow \frac{-3p(-1)^2 - q(-4)}{q(-1) + 6(-4)} = \frac{26}{19} \quad (1)$$

$$p = 2 \quad q = -5 \quad (1)$$

$$\frac{-3p + 4q}{-q - 24} = \frac{26}{19}$$

$$19(-3p + 4q) = 26(-q - 24)$$

$$-57p + 76q = -26q - 624$$

$$624 = 57p - 102q \quad (2)$$

(1)

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