Questions

Q1.

The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

(Total for question = 13 marks)

Q2.

The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point P (–1, – 4) lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

(Total for question = 9 marks)

Q3.

The curve C has parametric equations

$x = \sin 2\theta \qquad y = \text{cosec}^3\theta \qquad 0 < \theta < \frac{\pi}{2}$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

(Total for question = 6 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx}=\frac{\sqrt{3}\sin 2t}{\sin t}\quad\left(=2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t=\frac{2\pi}{3}$ in $\frac{dy}{dx}=\frac{\sqrt{3}\sin 2t}{\sin t}=\left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal $=-\frac{1}{dy/dx}=\left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P=\left(-1,-\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y+\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x-2\sqrt{3}y-1=0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$	M1	3.1a
	Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t-4\cos t-5=0$	A1	1.1b
	Finds $\cos t=\frac{5}{6},\cancel{-\frac{1}{2}}$	M1	2.4
	Substitutes their $\cos t=\frac{5}{6}$ into $x=2\cos t,\ \ y=\sqrt{3}\cos 2t$,	M1	1.1b
	$Q=\left(\frac{5}{3},\frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Notes:
(a) **M1:** Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t \cos t}{\sin t}$ **A1:** Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$
(b) **M1:** For substituting $t = \frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t **M1:** Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l. **B1:** States or uses (in their tangent or normal) that $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$ **M1:** Uses their numerical value of $-1\Big/\frac{dy}{dx}$ with their $\left(-1, -\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P **A1*:** This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$
(c) **M1:** For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation in t. Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form $y = Ax^2 + B$. **M1:** Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$ In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get an equation in just one variable **A1:** For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$ Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$ **M1:** Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P. **M1:** Substitutes their $\cos t = \frac{5}{6}$ or their $t = \arccos\left(\frac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ If a value of x or y has been found it is for finding the other coordinate. **A1:** $Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$. Allow $x = \frac{5}{3}, y = \frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Q2.

Question	Scheme	Marks	AOs
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(3y^2)=6y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}(qxy)=qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy$	M1	2.1
	$3px^2+qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$(qx+6y)\frac{\mathrm{d}y}{\mathrm{d}x}=-3px^2-qy\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$	dM1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3+q(-1)(-4)+3(-4)^2=26$	M1	1.1b
	$19x+26y+123=0\Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2-q(-4)}{q(-1)+6(-4)}=\frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2+q(-4)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624\Rightarrow p=\ldots,q=\ldots$	dM1	1.1b
	$p=2,\ q=-5$	A1	1.1b
		(5)	
			(9 marks)

Notes
(a)
M1: For selecting the appropriate method of differentiating:
Allow this mark for either $3y^2 \to \alpha y\frac{dy}{dx}$ or $qxy \to \alpha x\frac{dy}{dx} + \beta y$
A1: Fully correct differentiation. Ignore any spurious $\frac{dy}{dx} = ...$
dM1: A valid attempt to make $\frac{dy}{dx}$ the subject with 2 terms only in $\frac{dy}{dx}$ coming from qxy and $3y^2$
Depends on the first method mark.
A1: Fully correct expression
(b)
M1: Uses $x = -1$ and $y = -4$ in the equation of C to obtain an equation in p and q
B1: Deduces the correct gradient of the given normal.
This may be implied by e.g.
$19x + 26y + 123 = 0 \Rightarrow y = -\frac{19}{26}x + ... \Rightarrow$ Tangent equation is $y = \frac{26}{19}x + ...$
M1: Fully correct strategy to establish an equation connecting p and q using $x = -1$ and $y = -4$ in their $\frac{dy}{dx}$ and the gradient of the normal. E.g. $(a) = -1 \div \text{their} -\frac{19}{26}$ or $-1 \div (a) = \text{their} -\frac{19}{26}$
dM1: Solves simultaneously to obtain values for p and q.
Depends on both previous method marks.
A1: Correct values
Note that in (b), attempts to form the equation of the normal in terms of p and q and then compare coefficients with $19x + 26y + 123 = 0$ score no marks. If there is any doubt use Review.

Q3.

Question	Scheme	Marks	AOs
(a)	$y = \text{cosec}^3\theta \Rightarrow \frac{dy}{d\theta} = -3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	1.1b
	$\frac{dy}{dx} = \frac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \text{cosec}^3\theta = 8 \Rightarrow \sin^3\theta = \frac{1}{8} \Rightarrow \sin\theta = \frac{1}{2}$	M1	3.1a
	$\theta = \frac{\pi}{6} \Rightarrow \frac{dy}{dx} = \frac{-3\text{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)} = \ldots$ **or** $\sin\theta = \frac{1}{2} \Rightarrow \frac{dy}{dx} = \frac{\frac{-3}{\sin^3\theta} \times \frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)} = \frac{-3\times 8\times\frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1	2.1
	$= -24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

Notes
(a)   B1: Correct expression for $\frac{dy}{d\theta}$ seen or implied in any form e.g. $\frac{-3\cos\theta}{\sin^4\theta}$ M1: Obtains $\frac{dx}{d\theta} = k\cos 2\theta$ or $\alpha\cos^2\theta + \beta\sin^2\theta$ (from product rule on $\sin\theta\cos\theta$)   and attempts $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ A1: Correct expression in any form.   May see e.g. $\frac{-3\cos\theta}{2\sin^4\theta\cos 2\theta}$, $-\frac{3}{4\sin^4\theta\cos\theta - 2\sin^3\theta\tan\theta}$
(b)   M1: Recognises the need to find the value of $\sin\theta$ or θ when $y = 8$ and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta = \frac{1}{2}$ or e.g. $\theta = \frac{\pi}{6}$ or 30°.   M1: Uses their value of $\sin\theta$ or θ in their $\frac{dy}{dx}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\frac{dy}{dx}$. May be implied by a correct exact answer.   If no working is shown but an exact answer is given you may need to check that this follows their $\frac{dy}{dx}$.   A1: Deduces the correct gradient