Questions

Q1.

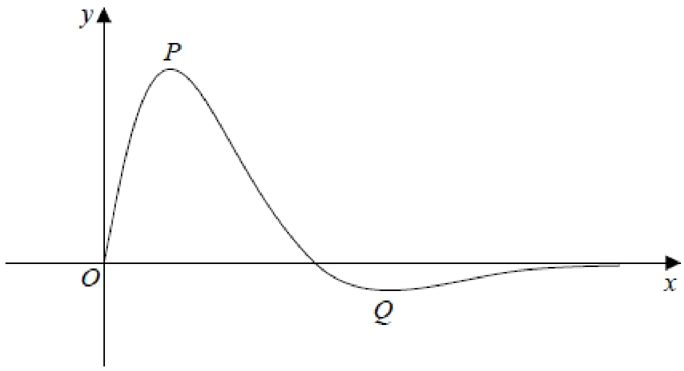

Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = f(2x)$.
(ii) $y = 3 - 2f(x)$.

(4)

(Total for question = 8 marks)

Q2.

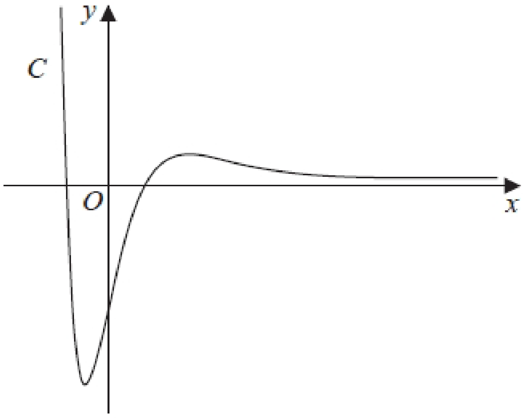

Figure 2

Figure 2 shows a sketch of the curve *C* with equation $y = f(x)$ where

$$f(x) = 4(x^2 - 2)e^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $f'(x) = 8(2 + x - x^2)e^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of *C*.

(3)

The function g and the function h are defined by

$$g(x) = 2f(x) \qquad x \in \mathbb{R}$$

$$h(x) = 2f(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

(Total for question = 9 marks)

Q3.

A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

(Total for question = 10 marks)

Q4.

The curve *C* has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that *C* has a point of inflection at $x = \sqrt[4]{27}$

(3)

(Total for question = 7 marks)

Q5.

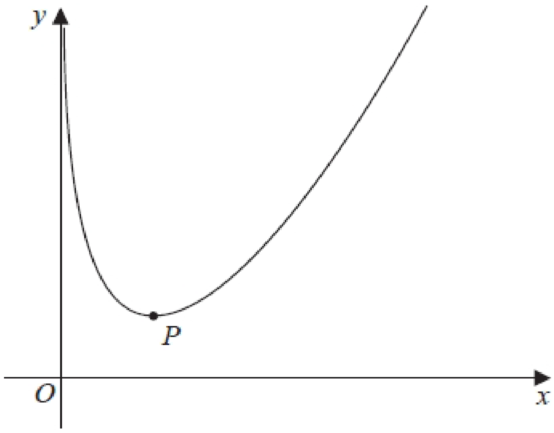

Figure 1

Figure 1 shows a sketch of the curve *C* with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point *P*, shown in Figure 1, is the minimum turning point on *C*.

(b) Show that the *x* coordinate of *P* is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,
(ii) the *x* coordinate of *P* to 5 decimal places.

(3)

(Total for question = 10 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$f'(x)=\dfrac{e^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $f'(x)=0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x=\sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x=\sqrt{2}$ and attempts to find the 2nd solution	M1	3.1a
	$x=1.02$	A1	1.1b
	(ii) Solves $\tan 2x=\sqrt{2}$ and attempts to find the 1st solution	M1	3.1a
	$x=0.478$	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts to differentiate by using the quotient rule with $u=4\sin 2x$ and $v=e^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u=4\sin 2x$ and $v=e^{1-\sqrt{2}x}$

A1: For achieving a correct $f'(x)$. For the product rule

$f'(x)=e^{1-\sqrt{2}x}\times 8\cos 2x+4\sin 2x\times -\sqrt{2}e^{1-\sqrt{2}x}$

M1: This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$

A1*: Proceeds to $\tan 2x=\sqrt{2}$. This is a given answer.

(b) (i)

M1: Solves $\tan 4x=\sqrt{2}$ attempts to find the 2nd solution. Look for $x=\dfrac{\pi+\arctan\sqrt{2}}{4}$

Alternatively finds the 2nd solution of $\tan 2x=\sqrt{2}$ and attempts to divide by 2

A1: Allow awrt $x=1.02$. The correct answer, with no incorrect working scores both marks

(b)(ii)

M1: Solves $\tan 2x=\sqrt{2}$ attempts to find the 1st solution. Look for $x=\dfrac{\arctan\sqrt{2}}{2}$

A1: Allow awrt $x=0.478$. The correct answer, with no incorrect working scores both marks

Q2.

Question	Scheme	Marks	AOs
(a)	$f(x)=4(x^2-2)e^{-2x}$		
	Differentiates to $e^{-2x}\times 8x+4(x^2-2)\times -2e^{-2x}$	M1 A1	1.1b 1.1b
	$f'(x)=8e^{-2x}\{x-(x^2-2)\}=8(2+x-x^2)e^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $f'(x)=0$ $x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $(-1,-4e^2)$ and $(2,8e^{-4})$	A1	1.1b
		(3)	
(c)	(i) Range $[-8e^2,\infty)$ o.e. such as $g(x)\geqslant -8e^2$	B1ft	2.5
	(ii) For • Either attempting to find $2f(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times "8e^{-4}"-3$ and identifying this as the **upper bound**	M1	3.1a
	Range $[-19,16e^{-4}-3]$	A1	1.1b
		(3)	
			(9 marks)
Notes:			

(a)

M1: Attempts the product rule and uses $e^{-2x}\to ke^{-2x}$, $k\neq 0$

If candidate states $u=4(x^2-2), v=e^{-2x}$ with $u'=....,v'=...e^{-2x}$ it can be implied by their $vu'+uv'$

If they just write down an answer without working award for $f'(x)=pxe^{-2x}\pm q(x^2-2)e^{-2x}$

They may multiply out first $f(x)=4x^2e^{-2x}-8e^{-2x}$. Apply in the same way condoning slips

Alternatively attempts the quotient rule on $f(x)=\frac{u}{v}=\frac{4(x^2-2)}{e^{2x}}$ with $v'=ke^{2x}$ and $f'(x)=\frac{vu'-uv'}{v^2}$

A1: A correct $f'(x)$ which may be unsimplified.

Via the quotient rule you can award for $f'(x)=\frac{8xe^{2x}-8(x^2-2)e^{2x}}{e^{4x}}$ o.e.

A1*: Proceeds correctly to given answer showing all necessary steps.

The $f'(x)$ or $\frac{dy}{dx}$ must be present at some point in the solution

This is a "show that" question and there must not be any errors. All bracketing must be correct.

Allow a candidate to move from the **simplified** unfactorised answer of $f'(x)=8xe^{-2x}-8(x^2-2)e^{-2x}$

to the given answer in one step.

Do not allow it from an **unsimplified** $f'(x) = 4 \times 2xe^{-2x} + 4(x^2 - 2) \times -2e^{-2x}$

Allow the expression / bracketed expression to be written in a different order.

So, for example, $8(x - x^2 + 2)e^{-2x}$ is OK

(b)

B1: States or implies $x = -1, 2$ (as the roots of $f'(x) = 0$)

M1: Substitutes one x value of their solution to $f'(x) = 0$ in f(x) to find a y value.

Allow decimals here (3sf). FYI, to 3 sf, $-4e^2 = -29.6$ and $8e^{-4} = 0.147$

Some candidates just write down the x coordinates but then go on in part (c) to find the ranges using the y coordinates. Allow this mark to be scored from work in part (c)

A1: Obtains $(-1, -4e^2)$ and $(2, 8e^{-4})$ as the stationary points. This must be scored in (b). Remember to isw after a correct answer. Allow these to be written separately. E.g. $x = -1$, $y = -4e^2$

Extra solutions, e.g. from $x = 0$ will be penalised on this mark.

(c)(i)

B1ft: For a correct range written using correct notation.

Follow through on 2 $\times$ their minimum "y" value from part (b), providing it is negative.

Condone a decimal answer if this is consistent with their answer in (b) to 3sf or better.

Examples of correct responses are $[-8e^2, \infty)$, $g \geqslant -8e^2$, $y \geqslant -8e^2$, $\{q \in \mathbb{R}, q \geqslant -8e^2\}$

(c)(ii)

M1: See main scheme. Follow through on $2 \times$ their "$8e^{-4}$" $- 3$ for the upper bound.

A1: Range $[-19, 16e^{-4} - 3]$ o.e. such as $-19 \leqslant y \leqslant 16e^{-4} - 3$ but must be exact

Q3.

Question	Scheme	Marks	AOs
	$N=\dfrac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0\ ;\quad \dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) Way 1	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)\ \left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1	2.1
		A1	1.1b
	$\dfrac{N(300-N)}{1200}=\dfrac{\left(\dfrac{900}{3+7e^{-0.25t}}\right)\left(300-\dfrac{900}{3+7e^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS}=\dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e., $\text{RHS}=\dfrac{900(300(3+7e^{-0.25t})-900)}{1200(3+7e^{-0.25t})^2}=\dfrac{1575e^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. and states hence $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N=150$ (can be implied)	B1	2.2a
	so $150=\dfrac{900}{3+7e^{-0.25T}}\Rightarrow e^{-0.25T}=\dfrac{3}{7}$	M1	3.4
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1	1.1b
		A1	1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Notes for Question	
(b)	
M1:	Attempts to differentiate using • the chain rule to give $\frac{dN}{dt} = \pm Ae^{-0.25t}(3+7e^{-0.25t})^{-2}$ or $\frac{\pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ o.e. • the quotient rule to give $\frac{dN}{dt} = \frac{(3+7e^{-0.25t})(0) \pm Ae^{-0.25t}}{(3+7e^{-0.25t})^2}$ • implicit differentiation to give $N(3+7e^{-0.25t}) = 900 \Rightarrow (3+7e^{-0.25t})\frac{dN}{dt} \pm ANe^{-0.25t} = 0$, o.e. where $A \neq 0$
Note:	Condone a slip in copying $(3+7e^{-0.25t})$ for the M mark
A1:	A correct differentiation statement
Note:	Implicit differentiation gives $(3+7e^{-0.25t})\frac{dN}{dt} - 1.75Ne^{-0.25t} = 0$
dM1:	**Way 1:** Complete attempt, by eliminating t, to form an equation linking $\frac{dN}{dt}$ and N only **Way 2:** Complete substitution of $N = \frac{900}{3+7e^{-0.25t}}$ into $\frac{dN}{dt} = \frac{N(300-N)}{1200}$
Note:	**Way 1:** e.g. substitutes $3+7e^{-0.25t} = \frac{900}{N}$ and $e^{-0.25t} = \frac{900}{N}$ or substitutes $e^{-0.25t} = \frac{\frac{900}{N}-3}{7}$ into their $\frac{dN}{dt} = ...$ to form an equation linking $\frac{dN}{dt}$ and N
A1*:	**Way 1:** Correct algebra leading to $\frac{dN}{dt} = \frac{N(300-N)}{1200}$ * **Way 2:** See scheme
(c)	
B1:	Deduces or shows that $\frac{dN}{dt}$ is maximised when $N = 150$
M1:	Uses the model $N = \frac{900}{3+7e^{-0.25t}}$ with their $N = 150$ and proceeds as far as $e^{-0.25T} = k,\ k > 0$ or $e^{0.25T} = k,\ k > 0$. Condone $t \equiv T$
dM1:	Correct method of using logarithms to find a value for T. Condone $t \equiv T$
A1:	see scheme
Note:	$\frac{d^2N}{dt^2} = \frac{dN}{dt}\left(\frac{300}{1200} - \frac{2N}{1200}\right) = 0 \Rightarrow N = 150$ is acceptable for B1
Note:	Ignore units for T
Note:	Applying $300 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ or $0 = \frac{900}{3+7e^{-0.25t}} \Rightarrow t = ...$ is M0 dM0 A0
Note:	M1 dM1 can only be gained in (c) by using an N value in the range $90 < N < 300$
(d)	
B1:	300 (or accept 299)

Question	Scheme	Marks	AOs
	$N=\frac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1},\ t\in\mathbb{R},\ t\geq 0\ ;\ \frac{dN}{dt}=\frac{N(300-N)}{1200}$		
(b) Way 3	$\int\frac{1}{N(300-N)}\,dN=\int\frac{1}{1200}\,dt$	M1	2.1
	$\int\frac{1}{300}\left(\frac{1}{N}+\frac{1}{300-N}\right)dN=\int\frac{1}{1200}\,dt$ $\frac{1}{300}\ln N-\frac{1}{300}\ln(300-N)=\frac{1}{1200}t\ \{+c\}$	A1	1.1b
	$\{t=0,\ N=90\Rightarrow\}\ c=\frac{1}{300}\ln(90)-\frac{1}{300}\ln(210)\Rightarrow c=\frac{1}{300}\ln\left(\frac{3}{7}\right)$ $\frac{1}{300}\ln N-\frac{1}{300}\ln(300-N)=\frac{1}{1200}t+\frac{1}{300}\ln\left(\frac{3}{7}\right)$ $\ln N-\ln(300-N)=\frac{1}{4}t+\ln\left(\frac{3}{7}\right)$ $\ln\left(\frac{N}{300-N}\right)=\frac{1}{4}t+\ln\left(\frac{3}{7}\right)\Rightarrow\frac{N}{300-N}=\frac{3}{7}e^{\frac{1}{4}t}$	dM1	2.1
	$7N=3e^{\frac{1}{4}t}(300-N)\Rightarrow 7N+3Ne^{\frac{1}{4}t}=900e^{\frac{1}{4}t}$ $N(7+3e^{\frac{1}{4}t})=900e^{\frac{1}{4}t}\Rightarrow N=\frac{900e^{\frac{1}{4}t}}{7+3e^{\frac{1}{4}t}}\Rightarrow N=\frac{900}{3+7e^{-0.25t}}$ *	A1*	1.1b
		(4)	
(b) Way 4	$N(3+7e^{-0.25t})=900\Rightarrow e^{-0.25t}=\frac{1}{7}\left(\frac{900}{N}-3\right)\Rightarrow e^{-0.25t}=\frac{900-3N}{7N}$ $\Rightarrow t=-4\left(\ln(900-3N)-\ln(7N)\right)$	M1	2.1
	$\Rightarrow\frac{dt}{dN}=-4\left(\frac{-3}{900-3N}-\frac{7}{7N}\right)$	A1	1.1b
	$\frac{dt}{dN}=4\left(\frac{1}{300-N}+\frac{1}{N}\right)\Rightarrow\frac{dt}{dN}=4\left(\frac{N+300-N}{N(300-N)}\right)$	dM1	2.1
	$\frac{dt}{dN}=\left(\frac{1200}{N(300-N)}\right)\Rightarrow\frac{dN}{dt}=\frac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	

Notes for Question Continued	
(b) Way 3	
M1:	Separates the variables, an attempt to form and apply partial fractions and integrates to give ln terms $=kt\ \{+c\},\ k\neq 0$, with or without a constant of integration c
A1:	$\frac{1}{300}\ln N-\frac{1}{300}\ln(300-N)=\frac{1}{1200}t\ \{+c\}$ or equivalent with or without a constant of integration c
dM1:	Uses $t=0,\ N=90$ to find their constant of integration and obtains an expression of the form $\lambda e^{\frac{1}{4}t}=f(N);\ \lambda\neq 0$ or $\lambda e^{-\frac{1}{4}t}=f(N);\ \lambda\neq 0$
A1*:	Correct manipulation leading to $N=\frac{900}{3+7e^{-0.25t}}$ *
(b) Way 4	
M1:	Valid attempt to make t the subject, followed by an attempt to find two ln derivatives, condoning sign errors and constant errors.
A1:	$\frac{dt}{dN}=-4\left(\frac{-3}{900-3N}-\frac{7}{7N}\right)$ or equivalent
dM1:	Forms a common denominator to combine their fractions
A1*:	Correct algebra leading to $\frac{dN}{dt}=\frac{N(300-N)}{1200}$ *

Q4.

Question	Scheme	Marks	AOs
(a)	$x^2 \tan y = 9 \Rightarrow 2x\tan y + x^2 \sec^2 y \dfrac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-2x \times \dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-18\times(x^4+81)-(-18x)(4x^3)}{(x^4+81)^2} = \dfrac{54(x^4-27)}{(x^4+81)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y \dfrac{dy}{dx}$ or $\cot y \to -\mathrm{cosec}^2 y \dfrac{dy}{dx}$

You may well see an attempt $\tan y = \dfrac{9}{x^2} \Rightarrow \sec^2 y \dfrac{dy}{dx} = ...$

When a candidate writes $x^2 \tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{\frac{1}{2}} y \to ...\tan^{\frac{3}{2}} y \sec^2 y$

A1: Correct differentiation $2x\tan y + x^2\sec^2 y \dfrac{dy}{dx} = 0$

Allow also $\sec^2 y \dfrac{dy}{dx} = -\dfrac{18}{x^3}$ or $2x = -9\mathrm{cosec}^2 y \dfrac{dy}{dx}$ amongst others

M1: Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$

A1*: Proceeds correctly to the given answer of $\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$

(b)

M1: Attempts to differentiate the given expression using the product or quotient rule.

For example look for a correct attempt at $\frac{vu'-uv'}{v^2}$ with $u=-18x, v=x^4+81, u'=\pm18, v'=...x^3$

If no method is seen or implied award for $\frac{\pm18\times(x^4+81)\pm18x(ax^3)}{(x^4+81)^2}$

Using the product rule award for $\pm18(x^4+81)^{-1}\pm18x(x^4+81)^{-2}\times cx^3$

A1: Correct **simplified** $\frac{d^2y}{dx^2}=\frac{54(x^4-27)}{(x^4+81)^2}$ o.e. such as $\frac{d^2y}{dx^2}=\frac{54x^4-1458}{(x^4+81)^2}$

Alternatively score for showing that when a correct (unsimplified) $\frac{d^2y}{dx^2}=0\Rightarrow x^4=27\Rightarrow x=\sqrt[4]{27}$

Or for substituting $x=\sqrt[4]{27}$ into an unsimplified but correct $\frac{d^2y}{dx^2}$ and showing that it is 0

A1: Correct explanation with a minimal conclusion and correct second derivative.

See scheme.

It can be also be argued from $x^4<27$, $x^4=27$ and $x^4>27$ provided the conclusion states that the point of inflection is at $x=\sqrt[4]{27}$

Alternatively substitutes values of x either side of $\sqrt[4]{27}$ and at $\sqrt[4]{27}$, into $\frac{d^2y}{dx^2}$, finds all three values and makes a minimal conclusion.

A different method involves finding $\frac{d^3y}{dx^3}$ and showing that $\frac{d^3y}{dx^3}\neq0$ and $\frac{d^2y}{dx^2}=0$ when $x=\sqrt[4]{27}$

FYI $\frac{d^3y}{dx^3}=\frac{23328x^3}{(x^4+81)^3}=0.219$ when $x=\sqrt[4]{27}$

Alternative part (a) using arctan

M1: Sets $y=\arctan\frac{9}{x^2}\rightarrow\frac{dy}{dx}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times...$ where ... could be 1

A2: $y=\arctan\frac{9}{x^2}\rightarrow\frac{dy}{dx}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times-\frac{18}{x^3}$

A1*: $\frac{dy}{dx}=\frac{1}{1+\frac{81}{x^4}}\times-\frac{18}{x^3}=\frac{-18x}{x^4+1}$ showing correct intermediate step and no errors.

Q5.

Question	Scheme	Marks	AOs
(a)	$\ln x \rightarrow \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0 \Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12} \Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	$x = 1.15650$	A1	2.2a
		(3)	
			(10 marks)

Notes:

(a)

B1: Differentiates $\ln x \rightarrow \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$:

Look for $\frac{4x^2+x}{2\sqrt{x}} \rightarrow ...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}}+...$ or $...+Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2+x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{dy}{dx}\right)=\frac{2\sqrt{x}(Ax+B)-(4x^2+x)Cx^{-\frac{1}{2}}}{(2\sqrt{x})^2}(A,B,C>0)$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu'-uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2+x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{dy}{dx}\right)=Ax^{-\frac{1}{2}}(Bx+C)+D(4x^2+x)x^{-\frac{3}{2}}(A,B,C>0)$

In general condone missing brackets for the M mark. If they quote $u=4x^2+x$ and $v=2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2+x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{dy}{dx}\right)=\frac{2\sqrt{x}(8x+1)-(4x^2+x)x^{-\frac{1}{2}}}{(2\sqrt{x})^2}$, $\frac{1}{2}x^{-\frac{1}{2}}(8x+1)-\frac{1}{4}(4x^2+x)x^{-\frac{3}{2}}$, $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{dy}{dx}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.
<u>There must be no errors e.g. missing brackets.</u>

(b)

M1: Sets $12x^2+x-16\sqrt{x}=0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}\Rightarrow x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12}\Rightarrow 12x^{\frac{3}{2}}=16-\sqrt{x}\Rightarrow 12x^2=16\sqrt{x}-x\Rightarrow 12x^2-16\sqrt{x}+x=0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by $\sqrt{x}$. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1=2$. This is implied by sight of $x_2=\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894

A1: Deduces that $x = 1.15650$