1. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \le t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)
(Total 10 marks)

2. The curve C has the equation

$$\cos 2x + \cos 3y = 1, \qquad -\frac{\pi}{4} \le x \le \frac{\pi}{4},\ 0 \le y \le \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(b) Find the value of y at P.

(3)

(c) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)
(Total 9 marks)

3. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

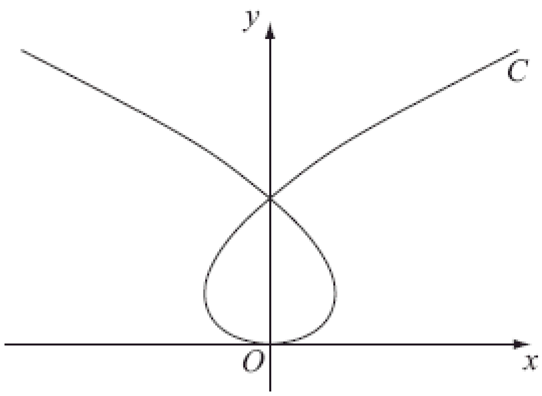
(5)

The point P on C has coordinates (0, 1).

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)
(Total 9 marks)

4.



The curve C shown above has parametric equations

$$x = t^3 - 8t,\ y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)
(Total 12 marks)

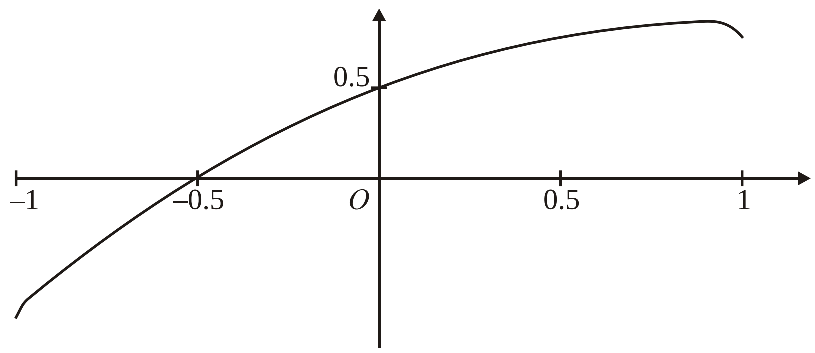
5. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 7 marks)

6.



The curve shown in the figure above has parametric equations

$$x = \sin t,\quad y = \sin\left(t + \tfrac{\pi}{6}\right),\qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

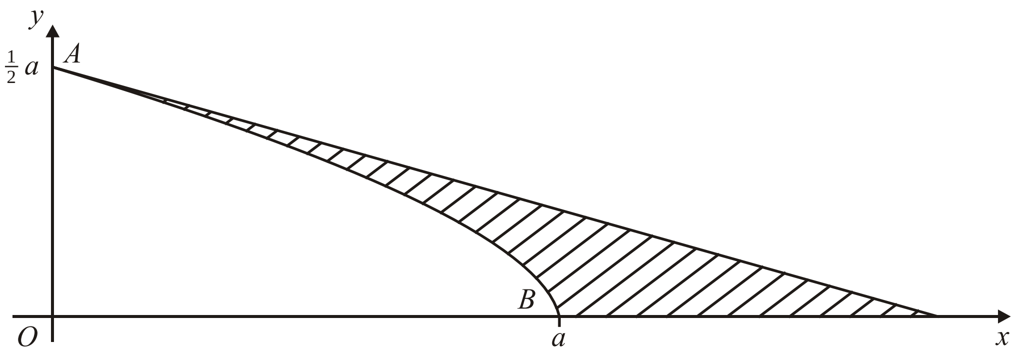
(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)},\quad -1 < x < 1$$

(3)
(Total 9 marks)

7.



The curve shown in the figure above has parametric equations

$$x = a\cos 3t,\ y = a\sin t, \quad 0 \leq t \leq \frac{\pi}{6}.$$

The curve meets the axes at points A and B as shown.

The straight line shown is part of the tangent to the curve at the point A.

Find, in terms of a,

(a) an equation of the tangent at A,

(6)

(b) an exact value for the area of the finite region between the curve, the tangent at A and the x-axis, shown shaded in the figure above.

(9)

(Total 15 marks)

8. A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point (1, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 7 marks)

9. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t.

(4)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(c) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined.

(4)
(Total 12 marks)

10. The curve C with equation $y = k + \ln 2x$, where k is a constant, crosses the x-axis at the point $A\left(\frac{1}{2e}, 0\right)$.

(a) Show that $k = 1$.

(2)

(b) Show that an equation of the tangent to C at A is $y = 2ex - 1$.

(4)

(c) Complete the table below, giving your answers to 3 significant figures.

x	1	1.5	2	2.5	3
1 + ln 2x		2.10		2.61	2.79

(2)

(d) Use the trapezium rule, with four equal intervals, to estimate the value of

$$\int_1^3 (1 + \ln 2x) \quad dx$$
.

(4)
(Total 12 marks)

11. $\mathrm{f}(x) = x + \dfrac{\mathrm{e}^x}{5}, \quad x \in \mathbb{R}.$

(a) Find f ′(*x*).

(2)

The curve *C*, with equation *y* = f(*x*), crosses the *y*-axis at the point *A*.

(b) Find an equation for the tangent to *C* at *A*.

(3)

(c) Complete the table, giving the values of $\sqrt{\left(x + \frac{\mathrm{e}^x}{5}\right)}$ to 2 decimal places.

x	0	0.5	1	1.5	2
$\sqrt{\left(x + \frac{\mathrm{e}^x}{5}\right)}$	0.45	0.91			

(2)

(d) Use the trapezium rule, with all the values from your table, to find an approximation for the value of

$$\int_0^2 \sqrt{\left(x + \frac{\mathrm{e}^x}{5}\right)} \, \mathrm{d}x$$
.

(4)
(Total 11 marks)

12. The curve C has equation $5x^2 + 2xy - 3y^2 + 3 = 0$. The point P on the curve C has coordinates (1, 2).

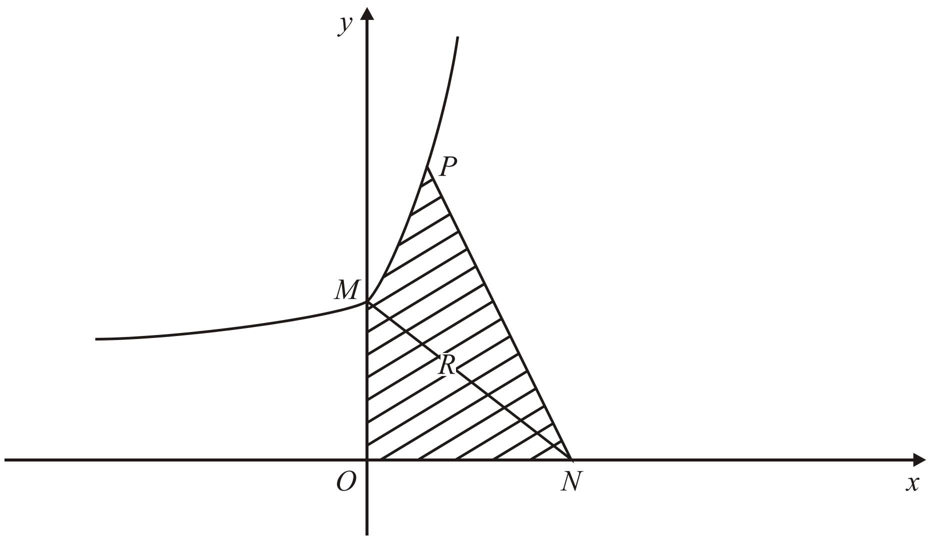
(a) Find the gradient of the curve at P.

(5)

(b) Find the equation of the normal to the curve C at P, in the form $y = ax + b$, where a and b are constants.

(3)
(Total 8 marks)

13.



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in the diagram above.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in the diagram above.

(c) Find the area of R, giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(7)
(Total 12 marks)

14. A curve has equation

$$x^3 - 2xy - 4x + y^3 - 51 = 0.$$

Find an equation of the normal to the curve at the point (4, 3), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 8 marks)

1. (a) $\frac{dx}{dt} = 2\sin t \cos t, \quad \frac{dy}{dt} = 2\sec^2 t$ B1 B1

$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t \cos t} \left(= \frac{1}{\sin t \cos^3 t} \right)$ or equivalent 4

(b) At $t = \frac{\pi}{3}, \ x = \frac{3}{4}, \ y = 2\sqrt{3}$ B1

$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin \frac{\pi}{3} \cos \frac{\pi}{3}} = \frac{16}{\sqrt{3}}$ A1

$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4} \right)$

$y = 0 \quad \Rightarrow \quad x = \frac{3}{8}$ A1 6

[10]

2. (a) $-2 \sin 2x - 3 \sin 3y \frac{dy}{dx} = 0$ A1

$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$ A1 3

(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6} \right) + \cos 3y = 1$

$\cos 3y = \frac{1}{2}$ A1

$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349 A1 3

(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9} \right)$, $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin \frac{\pi}{3}}{3\sin \frac{\pi}{3}} = -\frac{2}{3}$

$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6} \right)$

Leading to $6x + 9y - 2\pi = 0$ A1 3

[9]

3.	(a)	$e^{-2x}\frac{dy}{dx} - 2ye^{-2x} = 2 + 2y\frac{dy}{dx}$	A1 correct RHS	*	A1	
		$\frac{d}{dx}(ye^{-2x}) = e^{-2x}\frac{d}{dx} - 2ye^{-2x}$			B1	
		$(e^{-2x} - 2y)\frac{dy}{dx} = 2 + 2ye^{-2x}$		*		
		$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$			A1	5
	(b)	At P, $\frac{dy}{dx} = \frac{2 + 2e^{o}}{e^{o} - 2} = -4$				
		Using $mm' = -1$				
		$m' = \frac{1}{4}$				
		$y - 1 = \frac{1}{4}(x - 0)$				
		$x - 4y + 4 = 0$	or any integer multiple		A1	4
		Alternative for (a) differentiating implicitly with respect to y.				
		$e^{-2x} - 2ye^{-2x}\frac{dx}{dy} = 2\frac{dx}{dy} + 2y$	A1 correct RHS	*	A1	
		$\frac{d}{dy}(ye^{-2x}) = e^{-2x} - 2ye^{-2x}\frac{dx}{dy}$			B1	
		$(2 + 2ye^{-2x})\frac{dx}{dy} = e^{-2x} - 2y$		*		
		$\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2ye^{-2x}}$				
		$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$			A1	5

[9]

4.	(a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	$A(7, 1)$		B1	1
	(b)	$x = t^3 - 8t$, $y = t^2$,				
		$\frac{dx}{dt} = 3t^2 - 8, \frac{dy}{dt} = 2t$				

	$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$		
		Correct $\frac{dy}{dx}$	A1	
	At A, m(**T**) = $\frac{2(-1)}{3(-1)^2 - 8} = \frac{-2}{3-8} = \frac{-2}{-3} = \frac{2}{5}$	Substitutes for t to give any of the four underlined oe:		
	T: y – (their 1) = m_T(x – (their 7))	Finding an equation of a tangent with their point and their tangent gradient		
	or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$	or finds c and uses y = (their gradient)x + "c" .	dM1	
	Hence **T**: $y = \frac{2}{5}x - \frac{9}{5}$			
	gives **T**: $2x - 5y - 9 = 0$ **AG**		A1 cso	5
(c)	$2(t^3 - 8t) - 5t^2 - 9 = 0$	Substitution of both $x = t^3 - 8t$ and $y = t^2$ into **T**		
	$2t^3 - 5t^2 - 16t - 9 = 0$			
	$(t + 1)\{(2t^2 - 7t - 9) = 0\}$			
	$(t + 1)\{(t + 1)(2t - 9) = 0\}$	A realisation that $(t + 1)$is a factor.	dM1	
	$\{t = -1(\text{at } A)\ t = \frac{9}{2} \text{ at } B\}$	$t = \frac{9}{2}$	A1	
	$x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1	
	$y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25$ or awrt 20.3	One of either x or y correct.	A1	
		Both x and y correct.	A1	6
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$	awrt		

[12]

5. $\left\{\frac{\mathrm{d}y}{\mathrm{d}x}\right\}\times\quad 6x - 4y\frac{\mathrm{d}y}{\mathrm{d}x} + 2 - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ *or* $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$*.)*

Correct equation. A1

$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x+2}{4y+3}\right\}$

not necessarily required.

At (0, 1), $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{0+2}{4+3} = \frac{2}{7}$

Substituting x = 0 & y = 1 into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$; dM1

to give $\frac{2}{7}$ *or* $\frac{-2}{-7}$ A1 cso

Hence m(**N**) $= -\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ A1ftoe.

*Uses m(T) to 'correctly' find m(**N**).*
Can be ft from "their tangent gradient".

Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$
or: **N**: $y = -\frac{7}{2}x + 1$

$y - 1 = m(x - 0)$ + *with 'their tangent or normal gradient';*
or
uses y = mx + 1 with 'their tangent or normal gradient' ;

N: $7x + 2y - 2 = 0$

Correct equation in the form 'ax + by + c = 0', A1 oe
where a, b and c are integers. cso

[7]

Beware: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question.

So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions.
If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write y – 1 = ∞(x – 0). If they write, however, **N**: $x = 0$, then can score

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0,
and also obtains if they write y – 1 = 0(x – 0)or y = 1.

Beware: The final **cso** refers to the whole question.

Aliter Way 2

$\left\{\frac{\mathrm{d}x}{\mathrm{d}y}\right\} \times \quad 6x\frac{\mathrm{d}x}{\mathrm{d}y} - 4y + 2\frac{\mathrm{d}x}{\mathrm{d}y} - 3 = 0$

Differentiates implicitly to include either $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm 2\frac{\mathrm{d}x}{\mathrm{d}y}$

(ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} = \right)$*.)*

Correct equation. A1

$\left\{\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{4y+3}{6x+2}\right\}$

not necessarily required.

At (0, 1), $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{4+3}{0+2} = \frac{7}{2}$

Substituting x = 0 & y = 1 into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$*;* dM1

to give $\frac{7}{2}$ A1 cso

Hence m(**N**) = $\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ A1ftoe.

*Uses m(**T**) or $\frac{dx}{dy}$ to 'correctly' find m(**N**).*

*Can be ft using "**–1** . $\frac{dx}{dy}$ ".*

Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$
or **N**: $y = -\frac{7}{2}x + 1$

y – 1 = m(x – 0) with 'their tangent, $\frac{dx}{dy}$ or normal gradient';

or uses y = mx + 1 with 'their tangent, $\frac{dx}{dy}$ or normal gradient';

N: $7x + 2y - 2 = 0$

Correct equation in the form 'ax + by + c = 0', where a, b and c are integers. A1oe cso

[7]

Aliter Way 3

$2y^2 + 3y - 3x^2 - 2x - 5 = 0$

$\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$

$y = \sqrt{\frac{3x^2}{2} + x + \frac{49}{16}}) - \frac{3}{4}$

Differentiates using the chain rule;

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}}(3x + 1)$

Correct expression for $\frac{dy}{dx}$; A1 oe

At (0, 1),

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{4}{7}\right) = \frac{2}{7}$$

Substituting x = 0 into an equation involving $\frac{dy}{dx}$; dM1

to give $\frac{2}{7}$ *or* $\frac{-2}{-7}$ A1 cso

Hence m(**N**) = $-\frac{7}{2}$ A1ft

*Uses m(**T**) to 'correctly' find m(**N**).*
Can be ft from "their tangent gradient".

Either **N**: $y - 1 \;\; -\frac{7}{2}(x - 0)$
or **N**: $y \;\; = -\frac{2}{7}x + 1$

y – 1 = m(x – 0) with 'their tangent or normal gradient';
or uses y = mx + 1 with 'their tangent or normal gradient'

N: $7x + 2y - 2 = 0$ A1 oe

Correct equation in the form ' ax + by + c = 0',
where a, b and c are integers.

[7]

6. (a) x = sin t y = sin(t + $\frac{\pi}{6}$)

Attempt to differentiate both x and y wrt t to give two terms in cos

$$\frac{dx}{dt} = \cos t, \;\; \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$$ A1

Correct $\frac{dx}{dt}$ *and* $\frac{dy}{dt}$

When $t = \frac{\pi}{6}$,

$\frac{dy}{dx} = \underline{\frac{\cos(\frac{\pi}{6}+\frac{\pi}{6})}{\cos(\frac{\pi}{6})} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$ A1

Divides in correct way and substitutes for t to give any of the four underlined oe:
Ignore the double negative if candidate has differentiated sin → − cos

when $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{(\frac{1}{2}, \textit{awrt } 0.87)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - \frac{1}{2})}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses
y = (their gradient) x + "c". dM1
Correct <u>EXACT</u> equation of <u>tangent</u>
oe. A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or T: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$ 6

(b) $y = \sin(t + \frac{\pi}{6}) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$

Use of compound angle formula for sine.

Nb: $\sin^2 t + \cos^2 t = 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

$\therefore$ $x = \sin t$ gives $\cos t = \sqrt{(1 - x^2)}$

Use of trig identity to find cos t in terms of x or $\cos^2 t$ *in terms of x.*

$\therefore\ y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$

gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ AG A1 cso 3

Substitutes for sin *t,* cos $\frac{\pi}{6}$*, cost and sin* $\frac{\pi}{6}$ *to give y in terms of x.*

[9]

Aliter Way 2

(a) $x = \sin t$ $y = \sin(t + \frac{\pi}{6}) = \sin t + \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$

(Do not give this for part (b))

Attempt to differentiate x and y wrt t to give $\frac{dx}{dy}$ *in terms of cos and* $\frac{dy}{dt}$ *in the form ± a cost ± b sin t*

$\frac{dx}{dt} = \cos t$; $\frac{dy}{dt} = \cos t\cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$ A1

Correct $\frac{dx}{dt}$ *and* $\frac{dy}{dt}$

When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\dfrac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ A1

Divides in correct way and substitutes for t to give any of the four underlined oe

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses y = (their gradient)x + "c". dM1

Correct EXACT equation of <u>*tangent*</u> *oe.* A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$

Aliter Way 3

(a) $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$

Attempt to differentiate two terms using the chain rule for the second term.

Correct $\frac{dy}{dx}$ A1

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(1-(0.5)^2\right)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}$ A1

Correct substitution of x = $\frac{1}{2}$ *into a correct* $\frac{dy}{dx}$

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ *or* $(\frac{1}{2}, awrt\ 0.87)$

T: $y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - \frac{1}{2})$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses dM1
y = (their gradient) x + "c"

Correct EXACT equation of tangent oe. A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left\lfloor y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3} \right\rfloor$ 6

Aliter Way 2

(b) $x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$

Substitutes x = sin *t into the equation give in y.*

Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

Cost = $\sqrt{(1-\sin^2 t)}$

Use of trig identity to deduce that cos t = $\sqrt{(1-\sin^2 t)}$

gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$

Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin(t + \frac{\pi}{6})$ A1 cso 3

Using the compound angle formula to prove $y = \sin(t + \frac{\pi}{6})$

7. (a) $\frac{dx}{dt} = -3a\sin 3t$, $\frac{dy}{dt} = a\cos t$ therefore $\frac{dy}{dx} = \frac{\cos t}{-3\sin 3t}$ A1

When $x = 0$, $t = \frac{\pi}{6}$ B1

Gradient is $-\frac{\sqrt{3}}{6}$

Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$ A1 6

(b) Area beneath curve is $\int a\sin t\,(-3a\sin 3t)dt$

$= -\frac{3a^2}{2}\int(\cos 2t - \cos 4t)dt$

$\frac{3a^2}{2}[\frac{1}{2}\sin 2t - \frac{1}{4}\sin 4t]$ A1

Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$ A1

Area of triangle beneath tangent is $\frac{1}{2}\times\frac{a}{2}\times\sqrt{3}a = \frac{\sqrt{3}a^2}{4}$ A1

Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$ A1 9

N.B. The integration of the product of two sines is worth 3 marks
(lines 2 and 3 of to part (b))
If they use parts

$\int \sin t\sin 3t dt = -\cos t\sin 3t + \int 3\cos 3t\cos t dt$

$= -\cos t\sin 3t + 3\cos 3t\sin t + \int 9\sin 3t\sin t dt$

$8I = \cos t\sin 3t - 3\cos 3t\sin t$ A1

[15]

8. Differentiates

to obtain: $6x + 8y\frac{dy}{dx} - 2$,

.... $+ (6x\ \frac{dy}{dx} + 6y) = 0$ +(B1)

$$\left[\frac{dy}{dx} = \frac{2 - 6x - 6y}{6x + 8y}\right]$$

Substitutes $x = 1$, $y = -2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx} = -\frac{8}{10}$ A1

Uses line equation with numerical 'gradient' $y - (-2) =$ (their gradient) $(x - \mathbf{1})$ or finds c and uses $y =$ *(their gradient)* x + "c"

To give $5y + 4x + 6 = 0$ (or equivalent = 0) A1ft

[7]

9. (a) $\frac{dx}{dt} = -2\text{cosec}^2\, t$, $\frac{dy}{dt} = 4 \sin t \cos t$ both A1

$\frac{dy}{dx} = \frac{-2\sin t \cos t}{\text{cosec}^2 t}$ $(= -2\sin^3 t \cos t)$ A1 4

(b) At $t = \frac{\pi}{4}$, $x = 2$, $y = 1$ B1

both x and y

Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$

Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$ A1 4

Accept $x + 2y = 4$ *or any correct equivalent*

(c) Uses $1 + \cot^2 t = \text{cosec}^2\, t$, or equivalent, to eliminate t

$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ A1

correctly eliminates t

$y = \frac{8}{4 + x^2}$ cao A1

The domain ir $x\ldots0$ B1 4

Alternative for (c):

$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$

$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$ A1

Leading to $y = \frac{8}{4 + x^2}$ A1

[12]

10. (a) $0 = k + \ln 2\left(\frac{1}{2\mathrm{e}}\right) \Rightarrow 0 = k - 1 \Rightarrow k = 1$ (*) A1 2

(Allow also substituting k = 1and x = $\frac{1}{2e}$ into equation and showing y = 0 and substituting k = 1 and y = 0 and showing x = $\frac{1}{2e}$.)

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$ B1

At A gradient of tangent is $\frac{1}{1/2\mathrm{e}} = 2\mathrm{e}$

Equations of tangent: $y = 2\mathrm{e}\left(x - \frac{1}{2\mathrm{e}}\right)$

Simplifying to $y = 2\mathrm{e}x - 1$ (*) cso A1 4

(c) $y_1 = 1.69$, $y_2 = 2.39$ B1, B1 2

(d) $\int_1^3 (1 + \ln 2x)\mathrm{d}x \approx \frac{1}{2} \times \frac{1}{2} \times (...)$ B1

$\approx \ldots \times (1.69 + 2.79 + 2(2.10 + 2.39 + 2.61))$ ft their (c) A1ft

≈ 4.7 A1 4

accept 4.67

[12]

11. (a) Differentiating; $\mathbf{f'(x) = 1 + \frac{e^x}{5}}$ A1 2

	(b)	A: $\left(0, \frac{1}{5}\right)$	B1	
		Attempt at $y - f(0) = f'(0)x$;		
		$\boldsymbol{y - \frac{1}{5} = \frac{6}{5} x}$ or equivalent "one line" 3 termed equation	A1 ft	3
	(c)	**1.24, 1.55, 1.86**	B2(1,0)	2
	(d)	Estimate = $\frac{0.5}{2}$; (×) [(0.45 + 1.86) + 2(0.91 + 1.24 + 1.55)] B1	A1 ft	
		= **2.4275** $\left(\begin{matrix}\mathbf{2.428} \\ \mathbf{2.429}\end{matrix}, \ \mathbf{2.43}\right)$	A1	4

[11]

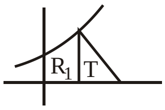
12.	(a)	$10x$, $+(2y + 2x\frac{dy}{dx})$, $-6y\frac{dy}{dx} = 0$	(B1), A1	
		At (1 , 2) $10 + (4 + 2\frac{dy}{dx}) - 12\frac{dy}{dx} = 0$		
		$\therefore \frac{dy}{dx} = \frac{14}{10} = 1.4$ or $\frac{7}{5}$ or $1\frac{2}{5}$	A1	5
	(b)	The gradient of the normal is $-\frac{5}{7}$		
		Its equation is $y - 2 = -\frac{5}{7}(x - 1)$		
		(allow tangent)		
		$y = -\frac{5}{7}x + 2\frac{5}{7}$ or $y = -\frac{5}{7}x + \frac{19}{7}$	A1cao	3

[8]

13.	(a)	M is (0, 7)	B1	
		$\frac{dy}{dx} = 2e^x$		
		Attempt $\frac{dy}{dx}$		
		$\therefore$ gradient of normal is $-\frac{1}{2}$		
		ft their y′(0) or $= -\frac{1}{2}$		
		(Must be a number)		
		$\therefore$ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $\underline{x + 2y - 14 = 0}$		

	$\underline{x + 2y = 14}$ o.e.	A1	4
(b)	$y = 0, x = 14 \therefore \underline{N \text{ is } (14, 0)}$ (*)	B1 cso	1

(c)



$\int (2e^x + 5)\,dx = [2e^x + 5x]$

some correct ∫

$R_1 = \int_0^{\ln 4} (2e^x + 5)\,dx = (2 \times 4 + 5 \ln 4) - (2 + 0)$

limits used

$= 6 + 5 \ln 4$	A1	
$T = \frac{1}{2} \times 13 \times (14 - \ln 4)$	B1	
Area of T		
$T = 13(7 - \ln 2)$; $R_1 = 6 + 10 \ln 2$	B1	
Use of ln 4 = 2ln 2		
$R = T + R_1$, $\underline{R = 97 - 3 \ln 2}$	A1	7

[12]

14. Differentiates w.r.t. x to give

$3x^2, -2x\frac{dy}{dx} + 2y, -4 + 3y^2\frac{dy}{dx} = 0$	B1, A1	
At (4, 3)		
$48 - (8y' + 6) - 4 + 27y' = 0$		
$\Rightarrow y' = -\frac{38}{19} = -2$	A1	
$\therefore$ Gradient of normal is $\frac{1}{2}$		
$\therefore y - 3 = \frac{1}{2}(x - 4)$		
i.e. $2y - 6 = x - 4$		
$x - 2y + 2 = 0$	A1	8

[8]

1. The majority of candidates knew how to tackle this question and solutions gaining all the method marks were common. However there were many errors of detail and only about 32% of the candidates gained full marks. In part (a), many candidates had difficult in differentiating $\sin^2 t$ and $2\tan t$. $2\tan t$ was more often differentiated correctly, possibly because the differential of $\tan t$ is given in the formula book, although 2 ln sec*t* or ln $\sec^2 t$ were often seen. Many could not differentiate $\sin^2 t$ correctly. $\cos^2 t$, 2 cos *t* and 2 sin *t* were all common. Nearly all candidates knew they had to divide $\frac{dy}{dt}$ by $\frac{dx}{dt}$, although there was some confusion in notation, with candidates mixing up their *x*s and *t*s. The majority knew how to approach part (b), finding the linear equation of the tangent to the curve at $\left(\frac{3}{4}, 2\sqrt{3}\right)$, putting $y = 0$ and solving for *x*. Some candidates used $y = 0$ prematurely and found the tangent to the curve at $\left(\frac{3}{4}, 0\right)$ rather than at $\left(\frac{3}{4}, 2\sqrt{3}\right)$.

2. As has been noted in earlier reports, the quality of work in the topic of implicit differentiation has improved in recent years and many candidates successfully differentiated the equation and rearranged it to find $\frac{dy}{dx}$. Some, however, forgot to differentiate the constant. A not infrequent, error was candidates writing $\frac{dy}{dx} = -2\sin 2x - 3\sin 3y \frac{dy}{dx}$ and then incorporating the superfluous $\frac{dy}{dx}$ on the left hand side of the equation into their answer. Errors like $\frac{dy}{dx}(\cos 3y)$ $= -\frac{1}{3}\sin 3y$.

were also seen. Part (b) was very well done. A few candidates gave the answer 20° , not recognising that the question required radians. Nearly all knew how to tackle part (c) although a few, as in Q2, spoilt otherwise completely correct solutions by not giving the answer in the form specified by the question.

3. As noted above work on this topic has shown a marked improvement and the median mark scored by candidates on this question was 8 out of 9. The only errors frequently seen were in differentiating $y e^{-2x}$ implicitly with respect to *x*. A few candidates failed to read the question correctly and found the equation of the tangent instead of the normal or failed to give their answer to part (b) in the form requested.

4. Part (a) was answered correctly by almost all candidates. In part (b), many candidates correctly applied the method of finding a tangent by using parametric differentiation to give the answer in the correct form. Few candidates tried to eliminate *t* to find a Cartesian equation for *C*, but these candidates were usually not able to find the correct gradient at *A*.

In part (c), fully correct solutions were much less frequently seen. A significant number of candidates were able to obtain an equation in one variable to score the first method mark, but were then unsure about how to proceed. Successful candidates mostly formed an equation in t, used the fact that $t + 1$ was a factor and applied the factor theorem in order for them to find t at the point B. They then substituted this t into the parametric equations to find the coordinates of B. Those candidates who initially formed an equation in y only went no further. A common misconception in part (c), was for candidates to believe that the gradient at the point B would be the same as the gradient at the point A and a significant minority of candidates attempted to solve $\frac{2t}{3t^2 - 8} = \frac{2}{5}$ to find t at the point B.

5. This question was successfully completed by the majority of candidates. Whilst many demonstrated a good grasp of the idea of implicit differentiation there were a few who did not appear to know how to differentiate implicitly. Candidates who found an expression for $\frac{dy}{dx}$ in terms of x and y, before substituting in values of x= 1 and y = 1, were prone to errors in manipulation. Some candidates found the equation of the tangent and a number of candidates did not give the equation of the normal in the requested form.

6. Part (a) was surprisingly well done by candidates with part (b) providing more of a challenge even for some candidates who had produced a perfect solution in part (a).

In part (a), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of t, although some candidates erroneously believed that differentiation of a sine function produced a negative cosine function. Other mistakes included a few candidates who either cancelled out "cos" in their gradient expression to give $\frac{t + \frac{\pi}{6}}{t}$ or substituted t = $\frac{\pi}{6}$ into their x and y expressions before proceeding to differentiate each with respect to t. Other candidates made life more difficult for themselves by expanding the y expression using the compound angle formula, giving them more work, but for the same credit. Many candidates were able to substitute t=$\frac{\pi}{6}$ into their gradient expression to give $\frac{1}{\sqrt{3}}$, but it was not uncommon to see some candidates who simplified $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ incorrectly to give $\sqrt{3}$ The majority of candidates wrote down the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and understood how to find the equation of the tangent using this point and their tangent gradient.

Whilst some candidates omitted part (b) altogether, most realised they needed to use the compound angle formula, though it was common to see that some candidates who believed that sin (t + $\frac{\pi}{6}$)could be rewritten as ' sin$t$ + sin$\frac{\pi}{6}$ '. Many candidates do not appreciate that a proof requires evidence, as was required in establishing that cos t = $\sqrt{1 - x^2}$, and so lost the final two marks. There were, however, a significant number of candidates who successfully obtained the required Cartesian equation.

7. This question proved a significant test for many candidates with fully correct solutions being rare. Many candidates were able to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$, although confusing differentiation with integration often led to inaccuracies. Some candidates attempted to find the equation of the tangent but many were unsuccessful because they failed to use $t = \frac{\pi}{6}$ in order to find the gradient as $-\frac{\sqrt{3}}{6}$.

Those candidates who attempted part (b) rarely progressed beyond stating an expression for the area under the curve. Some attempts were made at integration by parts, although very few candidates went further than the first line. It was obvious that most candidates were not familiar with integrating expressions of the kind $\int \sin at \sin bt \, dt$. Even those who were often spent time deriving results rather than using the relevant formula in the formulae book.

Those candidates who were successful in part (a) frequently went on to find the area of a triangle and so were able to gain at least two marks in part (b).

8. `This question was generally well answered with most candidates showing good skills in differentiating explicitly. Candidates who found an expression for $\frac{dx}{dy}$ in terms of x and y, before substituting in values, were more prone to errors in manipulation. Some candidates found the equation of the normal and a number of candidates did not give the equation of the tangent in the requested form. It was quite common to see such statements as $\frac{dx}{dy} = 6x + 8y\frac{dy}{dx} - 2 + \left(6x\frac{dy}{dx} + 6y\right) = 0$, but often subsequent correct working indicated that this was just poor presentation.

9. This proved a testing question and few could find both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correctly. A common error was to integrate x, giving $\frac{dx}{dt} = 2\ln(\sin t)$. Most knew, however, how to obtain $\frac{dy}{dx}$ from $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and were able to pick up marks here and in part (b). In part (b), the method for finding the equation of the tangent was well understood. Part (c) proved very demanding and only a minority of candidates were able to use one of the trigonometric forms of Pythagoras to eliminate t and manipulate the resulting equation to obtain an answer in the required form. Few even attempted the domain and the fully correct answer, x□ 0, was very rarely seen

10. In part (a), the log working was often unclear and part (b) also gave many difficulty. The differentiation was often incorrect. $\frac{1}{2x}$ was not unexpected but expressions like $x + \frac{1}{x}$ were also seen. Many then failed to substitute $x = \frac{1}{2e}$ into their $\frac{dy}{dx}$ and produced a non-linear tangent. Parts (c) and (d) were well done. A few did, however, give their answers to an inappropriate accuracy. As the table is given to 2 decimal places, the answer should not be given to a greater accuracy.

11. For many candidates this was a good source of marks. Even weaker candidates often scored well in parts (c) and (d). In part (a) there were still some candidates who were confused by the notation, f' often interpreted as f^{-1}, and common wrong answers to the differentiation were $\frac{e^x}{5}$ and $1 + e^x$. The most serious error, which occurred far too frequently, in part (b) was to have a variable gradient, so that equations such as $y - \frac{1}{5} = \left(1 + \frac{e^x}{5}\right)x$ were common. The normal, rather than the tangent, was also a common offering.

12. This was usually well done, but differentiation of a product caused problems for a number of candidates. Many still insisted on making $\frac{dy}{dx}$ the subject of their formula before substituting values for x and y. This often led to unnecessary algebraic errors.

13. Whilst the majority of answers to part (a) were fully correct, some candidates found difficulties here. A small number failed to find the coordinates of *M* correctly with (0, 5) being a common mistake. Others knew the rule for perpendicular gradients but did not appreciate that the gradient of a normal must be numerical. A few students did not show clearly that the gradient of the curve at $x = 0$ was found from the derivative, they seemed to treat $y = 2e^x + 5$ and assumed the gradient was always 2. Some candidates failed to obtain the final mark in this section because they did not observe the instruction that *a, b* and *c* must be integers.

For most candidates part (b) followed directly from their normal equation. It was disappointing that those who had made errors in part (a) did not use the absence of $n = 14$ here as a pointer to check their working in the previous part. Most preferred to invent all sorts of spurious reasons to justify the statement.

Many candidates set out a correct strategy for finding the area in part (c). The integration of the curve was usually correct but some simply ignored the lower limit of 0. Those who used the simple “half base times height” formula for the area of the triangle, and resisted the lure of their calculator, were usually able to complete the question. Some tried to find the equation of *PN* and integrate this but they usually made no further progress. The demand for exact answers proved more of a challenge here than in 6(c) but many candidates saw clearly how to simplify $2e^{\ln 4}$ and convert ln 4 into 2 ln 2 on their way to presenting a fully correct solution.

14. No Report available for this question.