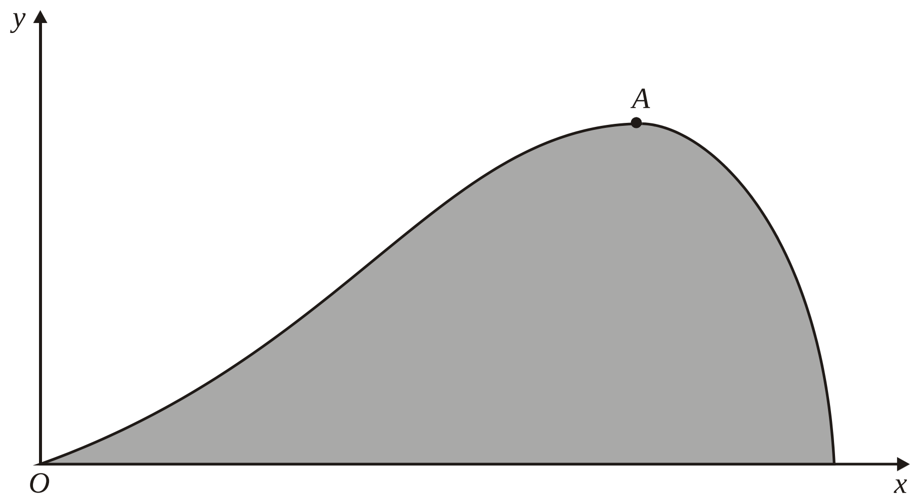
1. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(Total 7 marks)

2.



The diagram above shows a graph of $y = x\sqrt{\sin x}$, $0 < x < \pi$. The maximum point on the curve is A.

(a) Show that the x-coordinate of the point A satisfies the equation $2\tan x + x = 0$.

(4)

The finite region enclosed by the curve and the x-axis is shaded as shown in the diagram above.

A solid body S is generated by rotating this region through 2π radians about the x-axis.

(b) Find the exact value of the volume of S.

(7)
(Total 11 marks)

1. $2x + \left(2x\frac{dy}{dx} + 2y\right) - 6y\frac{dy}{dx} = 0$ (A1) A1

$\frac{dy}{dx} = 0 \Rightarrow x + y = 0$ (or equivalent)

Eliminating either variable and solving for at least one value of x or y.

$y^2 - 2y^2 - 3y^2 + 16 = 0$ or the same equation in x

$y = \pm 2$ or $x = \pm 2$ A1

$(2 - 2), (-2, 2)$ A1 7

Note: $\frac{dy}{dx} = \frac{x+y}{3y-x}$

Alternative:

$3y^2 - 2xy - (x^2 + 16) = 0$

$y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$

$\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3}.\frac{8x}{\sqrt{(16x^2 + 192)}}$ A1 ± A1

$\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$

$64x^2 = 16x^2 + 192$ A1

$x = \pm 2$ A1 7

$(2, -2), (-2, 2)$

[7]

2. (a) $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}}\cos x$ A1

At A $\sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}}\cos x = 0$ dM1

$\therefore \sin x + \frac{x}{2}\cos x = 0$ (essential to see intermediate line before given answer)

$\therefore 2\tan x + x = 0$ (*) A1 4

(b) $V = \pi\int y^2 dx = \pi\int x^2 \sin x dx$

$= \pi\left[-x^2\cos x + \int 2x\cos x dx\right]_0^\pi$ A1

$= \pi\left[-x^2\cos x + 2x\sin x - \int 2\sin x dx\right]_0^\pi$

$= \pi\left[-x^2\cos x + 2x\sin x + 2\cos x\right]_0^\pi$ A1

$= \pi[\pi^2 - 2 - 2]$

$= \pi[\pi^2 - 4]$ A1 7

[11]

1. Almost all candidates could start this question and the majority could differentiate implicitly correctly. This is an area of work which has definitely improved in recent years. Many, having found $\frac{\mathrm{d}y}{\mathrm{d}x}$, could not use it and it was disturbing to find a substantial number of students in this relatively advanced A2 module proceeding from $\frac{x+y}{3y-x}=0$ to $x + y = 3y - x$. Those who did obtain $y = -x$ often went no further but those who did could usually obtain both correct points, although extra incorrect points were often seen.

2. Most candidates made some attempt to differentiate $x\sqrt{\sin x}$, with varying degrees of success. $\sqrt{\sin x} + x\sqrt{\cos x}$ was the most common wrong answer. Having struggled with the differentiation, several went no further with this part. It was surprising to see many candidates with a correct equation who were not able to tidy up the √ terms to reach the required result.

Most candidates went on to make an attempt at $\int \pi y^2 \, \mathrm{d}x$. The integration by parts was generally well done, but there were many of the predictable sign errors, and several candidates were clearly not expecting to have to apply the method twice in order to reach the answer. A lot of quite good candidates did not get to the correct final answer, as there were a number of errors when substituting the limits.