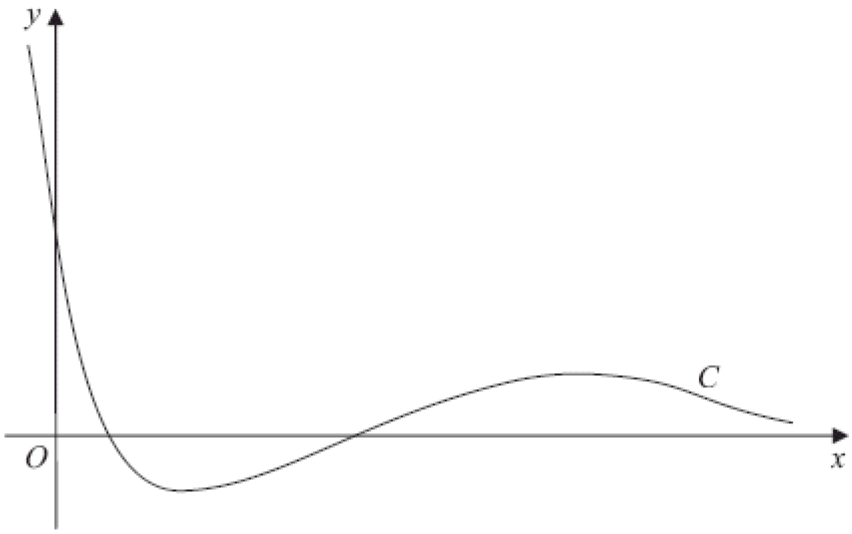
1.



The diagram above shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\frac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

(Total 12 marks)

2. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = \mathrm{e}^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)
(Total 11 marks)

3. A curve C has equation

$$y = x^2\mathrm{e}^x.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C.

(3)

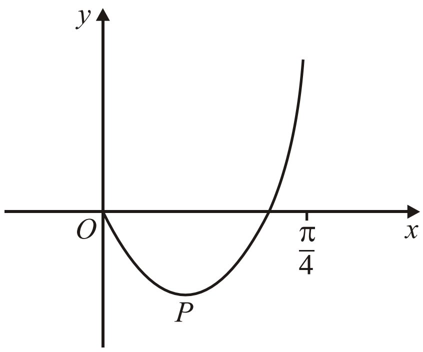
(c) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

(2)

(d) Determine the nature of each turning point of the curve C.

(2)
(Total 10 marks)

4.



The figure above shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leq x < \frac{\pi}{4}$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \qquad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

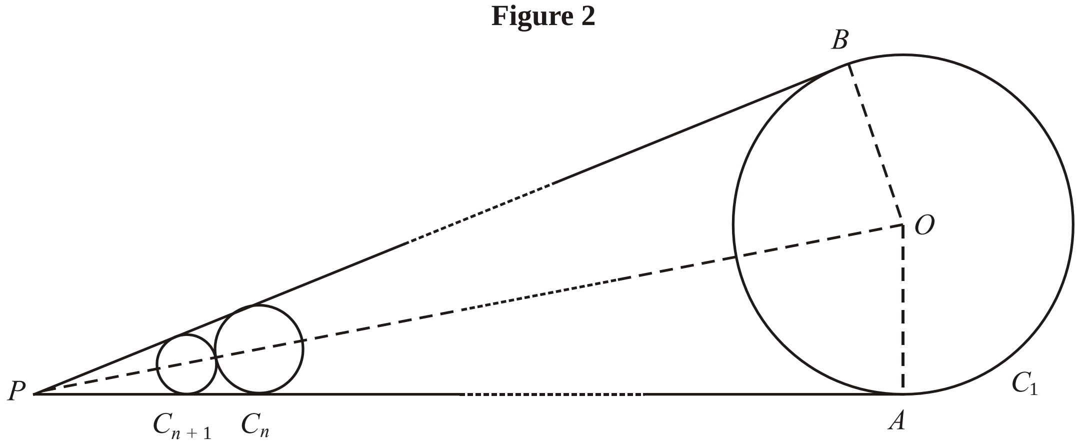
(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)
(Total 11 marks)

5.

Figure 2



The circle C_1 has centre O and radius R. The tangents AP and BP to C_1 meet at the point P and angle $APB = 2\alpha$, $0 < \alpha < \frac{\pi}{2}$. A sequence of circles C_1, ..., C_n, ... is drawn so that each new circle C_{n+1} touches each of C_n, AP and BP for $n = 1, 2, 3, ...$ as shown in the figure above. The centre of each circle lies on the line OP.

(a) Show that the radii of the circles form a geometric sequence with common ratio

$$\frac{1-\sin\alpha}{1+\sin\alpha}.$$

(5)

(b) Find, in terms of R and α, the total area enclosed by all the circles, simplifying your answer.

(3)

The area inside the quadrilateral $PAOB$, not enclosed by part of C_1 or any of the other circles, is S.

(c) Show that

$$S = R^2\left(\alpha + \cot\alpha - \frac{\pi}{4}\operatorname{cosec}\alpha - \frac{\pi}{4}\sin\alpha\right).$$

(5)

(d) Show that, as α varies,

$$\frac{dS}{d\alpha} = R^2 \cot^2\alpha \left(\frac{\pi}{4}\cos\alpha - 1\right).$$

(4)

(e) Find, in terms of R, the least value of S for $\frac{\pi}{6} \le \alpha \le \frac{\pi}{4}$.

(3)
(Total 20 marks)

6. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

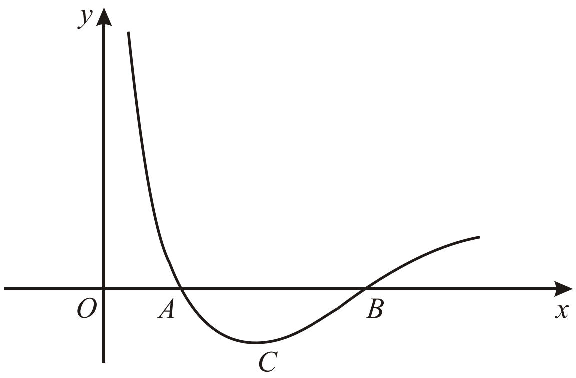
(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)
(Total 9 marks)

7.



$$f(x) = \frac{1}{2x} - 1 + \ln\frac{x}{2},\ x > 0.$$

The diagram above shows part of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points A and B, and has a minimum at the point C.

(a) Show that the x-coordinate of C is $\frac{1}{2}$.

(5)

(b) Find the y-coordinate of C in the form $k \ln 2$, where k is a constant.

(2)

(c) Verify that the x-coordinate of B lies between 4.905 and 4.915.

(2)

(d) Show that the equation $\frac{1}{2x} - 1 + \ln\frac{x}{2} = 0$ can be rearranged into the form $x = 2e^{\left(1 - \frac{1}{2x}\right)}$.

(2)

The x-coordinate of B is to be found using the iterative formula

$$x_{n+1} = 2e^{\left(1 - \frac{1}{2x_n}\right)}, \quad \text{with } x_0 = 5.$$

(e) Calculate, to 4 decimal places, the values of x_1, x_2 and x_3.

(2)
(Total 13 marks)

8. The curve C has equation $y = \dfrac{x}{4 + x^2}$.

(a) Use calculus to find the coordinates of the turning points of C.

(5)

Using the result $\dfrac{d^2y}{dx^2} = \dfrac{2x(x^2 - 12)}{(4 + x^2)^3}$, or otherwise,

(b) determine the nature of each of the turning points.

(3)

(c) Sketch the curve C.

(3)
(Total 11 marks)

9. The curve C has equation $y = f(x)$, where

$$f(x) = 3 \ln x + \frac{1}{x}, \quad x > 0.$$

The point P is a stationary point on C.

(a) Calculate the x-coordinate of P.

(4)

(b) Show that the y-coordinate of P may be expressed in the form $k - k \ln k$, where k is a constant to be found.

(2)

The point Q on C has x-coordinate 1.

(c) Find an equation for the normal to C at Q.

(4)

The normal to C at Q meets C again at the point R.

(d) Show that the x-coordinate of R

(i) satisfies the equation $6 \ln x + x + \frac{2}{x} - 3 = 0$,

(ii) lies between 0.13 and 0.14.

(4)
(Total 14 marks)

1.	(a)	Either $y = 2$ or $(0, 2)$	B1	1
	(b)	When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1	
		$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$		
		Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1	3

Note

If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark.

	(c)	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	A1 A1	3

Note

(their u')e^{-x} + $(2x^2 - 5x + 2)$(their v')
A1: Any one term correct.
A1: Both terms correct.

	(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$			
		$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$			
		$x = \frac{7}{2}, 1$		A1	
		When $x = \frac{7}{2}$, $y = 9e^{-\frac{7}{2}}$, when $x = 1$, $y = -e^{-1}$	dd	A1	5

Note

1st For setting their $\frac{dy}{dx}$ found in part (c) equal to 0.

2nd Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix.

3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part.

Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf.

Final A1: Both$\{x = 1\}$, $y = -e^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. **cao**

Note that both exact values of y are required.

[12]

2. (a) $y = \sec x = \dfrac{1}{\cos x}$

$= (\cos x)^{-1}$

$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ — $\dfrac{dy}{dx} = \pm 1((\cos x)^{-2}(-\sin x))$

$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$ — A1

$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} =$

$\underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$ — Convincing proof.

Must see both underlined steps. — A1 **AG** — 3

(b) $y = e^{2x}\sec 3x$

$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{array}\right\}$

Seen or implied

Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$

Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$ — A1

$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$ — Applies $vu' + uv'$ correctly for their u, u', v, v'

$2e^{2x}\sec 3x + 3e^{2x}\sec 3x\tan 3x$ — A1 **isw** — 4

(c) Turning point $\Rightarrow \dfrac{dy}{dx} = 0$

Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$ — Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels)

{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$,} — out at least e^{2x} from at least two terms.

giving $\tan 3x = -\frac{2}{3}$ — $\tan 3x = \pm k$; $k \neq 0$

$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\}$

$= -0.19600...$	Either awrt -0.196^c or awrt $-11.2°$	A1	
Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$ $= 0.812093... = 0.812$ (3sf)	0.812	A1 **cao**	4

Note

If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.

Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

[11]

3. (a) $\frac{dy}{dx} = x^2e^x + 2xe^x$ A1, A1 3

M for attempt at $f(x)g'(x) + f'(x)g(x)$
1st A1 for one correct, 2nd A1 for the other correct.
Note that x^2e^x on its own scores no marks

(b) If $\frac{dy}{dx} = 0$, $e^x(x^2 + 2x) = 0$ setting $(a) = 0$

$[e^x \neq 0]$ $x(x + 2) = 0$
$(x = 0)$ $x = -2$ A1
$x = 0, y = 0$ **and** $x = -2, y = 4e^{-2}$ $(= 0.54...)$ A1ft 3

1st A1 ($x = 0$) may be omitted, but for 2nd A1 both sets of coordinates needed; ft only on candidate's $x = -2$

(c) $\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$ $[= (x^2 + 4x + 2)e^x]$ A1 2

requires complete method for candidate's (a), result may be unsimplified for A1

(d) $x = 0$, $\frac{d^2y}{dx^2} > 0$ $(= 2)$ $x = -2$, $\frac{d^2y}{dx^2} < 0$ $[= -2e^{-2}$ $(= -0.270...)]$

Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b)

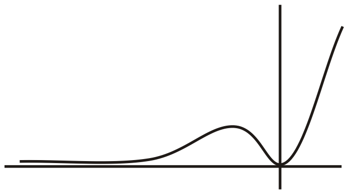
$\therefore$ minimum $\therefore$ maximum A1cso 2

Alt. (d) For

Evaluate, or state sign of, $\frac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b) or

Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or

Sketch curve



A1 is cso; $x = 0$, min. and $x = -2$, max and no correct working seen, or (in alternative)sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.

Need only consider the quadratic, as may assume $e^x > 0$.

If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

[10]

4. (a) Using product rule: $\frac{dy}{dx} = 2\tan 2x + 2(2x-1)\sec^2 2x$ A1 A1

Use of " $\tan 2x = \frac{\sin 2x}{\cos 2x}$ " and " $\sec 2x = \frac{1}{\cos 2x}$ "

$\left[= 2\frac{\sin 2x}{\cos 2x} + 2(2x-1)\frac{1}{\cos^2 2x}\right]$

Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions

$[\Rightarrow 2\sin 2x\cos 2x + 2(2x - 1) = 0]$

Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG A1 6

(b) $x_1 = 0.2670$, $x_2 = 0.2809$, $x_3 = 0.2746$, $x_4 = 0.2774$, A1 A1 3

for first correct application, first A1 for two correct, second A1 for all four correct

Max –1 deduction, if ALL correct to > 4 d.p. A0 A1

SC: degree mode: $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2

(c) Choose suitable interval for *k:* e.g. [0.2765, 0.2775]
and evaluate f(x) at these values

Show that $4k + \sin 4k - 2$ changes sign and deduction A1 2

[f(0.2765) = –0.000087.., f(0.2775) = +0.0057]

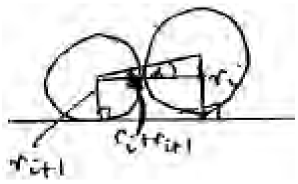
Continued iteration: (no marks in degree mode)

Some evidence of further iterations leading to 0.2765 or better

Deduction A1

[11]

5. (a)



Appropriate figure

$\Rightarrow \sin\alpha = \dfrac{r_i - r_{i+1}}{r_i + r_{i+1}}$ (exp for $\sin\alpha$) A1

$\therefore r_i + r_{i+1})\sin\alpha = r_i - r_{i+1}$ $\left(\dfrac{r_{i+1}}{r_i}\right)$

$\therefore$ ration of radii $= \dfrac{1-\sin\alpha}{1+\sin\alpha}$ * (=r) A1 c.s.o. 5

(b) Total area $= \pi R^2 + \pi r_2{}^2 + \pi r_3{}^2 + \ldots$

$= \pi R^2(1 + r^2 + r^4 + \ldots)$ (correct "r") B1

$= \dfrac{\pi R^2}{1-r^2} = \pi R^2 \dfrac{1}{1-\left(\dfrac{1-\sin\alpha}{1+\sin\alpha}\right)^2}$

$= \dfrac{\pi R^2(1+\sin\alpha)^2}{(1+\sin\alpha)^2 - (1-\sin\alpha)^2}$ $= \dfrac{\pi R^2(1+\sin\alpha)^2}{4\sin\alpha}$ A1 3

(c) Required area = 2 × Area Δ *POA* + Area major sector *AOB*

– Area found in (b)

Area Δ *POA* = $\frac{1}{2}R(R\cot\alpha)$ B1

$\angle POA = \frac{\pi}{2} - \alpha \quad \therefore$ angle of major sector to B $= \pi + 2\alpha$

$\therefore$ Area sector *AOB* = $\frac{1}{2}R^2(\pi + 2\alpha)$ A1

$\therefore$ Required area = $R^2(\cot\alpha + \frac{\pi}{2} + d - \frac{\pi}{4}\left(\frac{1+2\sin\alpha+\sin^2\alpha}{\sin\alpha}\right)$ A1 c.s.o. 5

$= R^2(\alpha + \cot\alpha - \frac{\pi}{4}\operatorname{cosec}\alpha - \frac{\pi}{4}\sin\alpha)$

(d) $\frac{ds}{d\alpha} = R^2\left(1 - \operatorname{cosec}^2\alpha + \frac{\pi}{4}\operatorname{cosec}\alpha\cot\alpha - \frac{\pi}{4}\cos\alpha\right)$ A1

$= R^2(-\cot^2\alpha + \frac{\pi}{4}\frac{\cos\alpha}{\sin\alpha} - \frac{\pi}{4}\cos\alpha)$

$= R^2(-\cot^2\alpha + \frac{\pi}{4}\cos\alpha(\operatorname{cosec}^2\alpha - 1))$

$= R^2\cot^2\alpha(\frac{\pi}{4}\cos\alpha - 1)$ A1 (c.s.o) 4

(use of $\cot^2\alpha = \operatorname{cosec}^{\alpha} - 1$) o.e.

(e) In the given range $R^2\cot^2\alpha > 0$

In the interval

$(0, \frac{\pi}{4})$; $\frac{\pi}{4}\cos\alpha - 1$ is a decreasing function ($\because$ $\cos\alpha$ is decreasing).

At $\alpha = 0$, $\frac{\pi}{4}\cos\alpha - 1 = \frac{\pi}{4} - 1 < 0$

$\therefore \frac{\pi}{4}\cos\alpha - 1 < 0$ in $(\frac{\pi}{6}, \frac{\pi}{4})$

$\therefore \frac{ds}{d\alpha} < 0$ throughout the interval (convincing argument)

$\therefore$ Least value of S occurs at $\alpha = \frac{\pi}{4}$ A1

Min S $= R^2\left(\frac{\pi}{4} + 1 - \frac{\pi}{4}.\sqrt{2} - \frac{\pi}{4}.\frac{1}{\sqrt{2}}\right)$

$= R^2\left(1-\frac{\pi}{4}\left(-1+\sqrt{2}+\frac{1}{\sqrt{2}}\right)\right)$ o.e. A1 3

[20]

6. (a) $f'(x) = 3\,e^x - \frac{1}{2x}$ M1A1A1 3

any evidence to suggest that tried to differentiate

(b) $3e^{\alpha} - \frac{1}{2\alpha} = 0$

Equating f′ (x) to zero

$\Rightarrow 6\alpha\, e^{\alpha} = 1 \Rightarrow \alpha = \frac{1}{6}e^{-\alpha}$ AG A1(cso) 2

(c) $x_1 = 0.0613\ldots$, $x_2 = 0.1568\ldots$, $x_3 = 0.1425\ldots$, $x_4 = 0.1445\ldots$ M1A1 2

at least x_1 correct, A1 all correct to 4 d.p.

(d) Using $f'(x)$ {$= 3\,e^x - \frac{1}{2x}$} with suitable interval
[e.g. f(0.14425) = – 0.007, f(0.14435) = +0.002(1)]
Both correct with concluding statement. A1 2

[9]

7. (a) $f'(x) = -\frac{1}{2x^2}; + \frac{1}{x}$ M1A1;A1

for evidence of differentiation. Final A – no extras

$f'(x) = 0 \Rightarrow \frac{-1+2x}{2x^2} = 0; \Rightarrow x = 0.5$ M1A1 (*) cso 5

(or subst x = 0.5)

(b) $y = 1 - 1 + \ln\left(\frac{1}{4}\right); = -2\ln 2$ A1 2

Sust 0.5 or their value for x in

(c) f(4.905) = < 0 (–0.000955), f(4.915) = > 0 (+ 0.000874)

evaluate

Change of sign indicates root between and correct values to 1 sf) A1 2

(d) $\frac{1}{2x} - 1 + \ln\left(\frac{x}{2}\right) = 0; \;\; \Rightarrow 1 - \frac{1}{2x} = \ln\left(\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2} = e^{\left(1 - \frac{1}{2x}\right)}; \Rightarrow x = 2e^{\left(1 - \frac{1}{2x}\right)}$ (*) (c.s.o.) A1 2

for use of e to the power on both sides

(e) $x_1 = 4.9192$ B1

$x_2 = 4.9111$, $x_3 = 4.9103$, B1 2

both, only lose one if not 4dp

[13]

8. (a) $\frac{dy}{dx} = \frac{(4x + x^2) - x(2x)}{(4 + x^2)^2}$ or (from product rule) $(4 + x^2)^{-1} - 2x^2(4 + x^2)^{-2}$ M1 A1

Solve $\frac{dy}{dx} = 0$ to obtain (2, ¼), and (–2, –¼) A1, A1 5

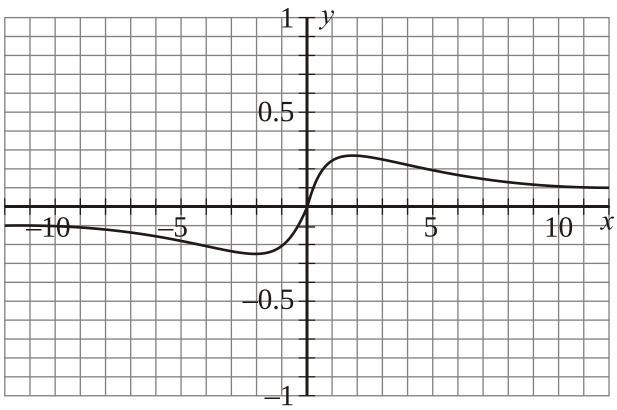
or (2 and –2 A1, full solution A1)

(b) When $x = 2$, $\frac{d^2y}{dx^2} = -0.0625 < 0$ thus maximum B1

Need numerical answers for

When $x = -2$, $\frac{d^2y}{dx^2} = 0.0625 > 0$ thus minimum. B1 3

(c)



Shape for $-2 \le x \le 2$ B1

Shape for $x > 2$ B1

Shape for $x < 2$ B1 3

[11]

9.	(a)	$f'(x) = \frac{3}{x} - \frac{1}{x^2}$	A1	
		$\frac{3}{x} - \frac{1}{x^2} = 0 \Rightarrow 3x^2 - x = 0 \Rightarrow x = \frac{1}{3}$	A1	4
	(b)	$y = 3\ln\left(\frac{1}{3}\right) + \frac{1}{\left(\frac{1}{3}\right)} = 3 - 3\ln 3$ ($k = 3$)	A1	2
	(c)	$x = 1 \Rightarrow y = 1$	B1	
		$f'(1) = 2 \Rightarrow m = -\frac{1}{2}$		
		$y - 1 = -\frac{1}{2}(x - 1)$ $\left(y = -\frac{x}{2} + \frac{3}{2}\right)$	A1	4
	(d)	(i) $-\frac{x}{2} + \frac{3}{2} = 3\ln x + \frac{1}{x}$		
		leading to $6\ln x + x + \frac{2}{3} - 3 = 0$ (*)	A1	
		cso		
		(ii) g(0.13) = 0.273…		
		g(0.14) = –0.370…		
		Both, accept one d.p.		
		Sign change (and continuity) $\Rightarrow$ root $\in$ (0.13, 0.14)	A1	4

[14]

1. This question was extremely well answered with 84% of candidates gaining at least 7 of the 12 marks available and about 42% gaining all 12 marks.

Nearly all candidates were successful in answering part (a). A few candidates were initially confused when attempting part (a) by believing that the curve met the y-axis when $y = 0$. These candidates quickly recovered and relabelled part (a) as their part (b) and then went onto to find in part (a) that when $x = 0$, $y = 2$. Therefore, for these candidates, part (b) was completed before part (a).

In part (b), some candidates chose to substitute $x = 2$ into $y = (2x^2 - 5x + 2)e^{-x}$ in order to confirm that $y = 0$ The majority of candidates, however, set $y = 0$ and solved the resulting equation to give both $x = 2$ and $x = 0.5$ Only a few candidates wrote that $x = 0$ is a solution of $e^{-x} = 0$.

In part (c), the product rule was applied correctly to $(2x^2 - 5x + 2)e^{-x}$ by a very high proportion of candidates with some simplifying the result to give $(-2x^2 + 9x - 7)e^{-x}$ Common errors included either e^{-x} being differentiated incorrectly to give e^{-x} or poor bracketing. The quotient rule was rarely seen, but when it was it was usually applied correctly.

In part (d), the majority of candidates set their $\frac{dy}{dx}$ in part (c) equal to 0, although a few differentiated again and set $\frac{d^2y}{dx^2} = 0$. At this stage, few candidates produced invalid logarithmic work and lost a number of marks. Some other candidates made bracketing and/or algebraic errors in simplifying their gradient function. Most candidates realised that they needed to factorise out e^{-x} and solve the resulting quadratic with many of them correctly finding both sets of coordinates. Some candidates did not give their y-coordinates in terms of e, but instead wrote the decimal equivalent.

2. In part (a), candidates used the quotient rule more often than a direct chain rule. The quotient rule was often spoiled by some candidates who wrote down that 1 was the derivative of 1. Another common error was for candidates to write down that $0 \times \cos x = \cos x$. Some proofs missed out steps and only fully convincing proofs gained all 3 marks with the final mark sometimes lost through lack of an explicit demonstration.

In part (b), many candidates differentiated e^{2x} correctly but common mistakes for the derivative of $\sec 3x$ were $\sec 3x \tan 3x$ or $3\sec x \tan x$. The product rule was applied correctly to $e^{2x}\sec 3x$ by a very high proportion of candidates, although occasionally some candidates applied the quotient rule to differentiate $\frac{e^2x}{\sec 3x}$. A few candidates applied the quotient rule to differentiate $e^{2x}\sec 3x$ when the product rule would have been correct.

Those candidates, who attempted to differentiate $e^{2x}\sec 3x$ in part (b), were able to set their $\frac{dy}{dx}$ equal to zero and factorise out at least e^{2x},with a significant number of candidates getting as far as $\tan 3x = \pm k$, $(k \neq 0)$, with some candidates giving k as –2. Many of the candidates who achieved $k = -\frac{2}{3}$, were able to find the correct answer for a of –0.196, although a few of them incorrectly stated a as 0.196. A surprising number of good candidates, having found the correct value for x, were then unable to correctly evaluate y. It was not required to prove the nature of the turning point, so it was a waste of several candidates' time to find an expression for the second derivative.

3. It was pleasing to see the majority of candidates gain full marks in part (a), and there was an impressive number who went on to produce a completely correct expression in part (c). In part (b) most candidates knew that they were required to solve $e^x(x^2 + 2x) = 0$, but often the solution $x = 0$ was omitted, or the coordinate**s** of the turning points were not given, or the y-coordinate for $x = 0$ was calculated to be 1.
The method mark in part (d) was often gained but to gain both marks required a correct conclusion and a substantially correct solution to the question.

4. The product rule is well known and was accurately applied by many candidates in part (a). Rather than changing tan $2x$ to $\frac{\sin 2x}{\cos 2x}$ and $\sec^2 2x$ to $\frac{1}{\cos^2 2x}$ some candidates used the identity $1 + \tan^2 2x = \sec 2x$ These candidates were rarely able to make progress beyond a few more lines of manipulation and such solutions were often abandoned. Algebraic manipulation was a problem for some candidates. Others never set $\frac{dy}{dx}$ equal to zero and incorrectly multiplied only one side of their equation by $\cos^2 2x$ rather than using a common denominator or stating that $\frac{dy}{dx} = 0$ before multiplying by $\cos^2 2x$ This part of the question asked candidates to show a given result and candidates did not always show sufficient steps in their work. Full marks were not awarded unless the 4 sin k part of the equation came from an intermediate result of 2 sin 2 k 2 cos k somewhere in the solution. Many correct solutions were seen in (b), although a few candidates were inaccurate when giving their answers to 4 decimal places. By far the most common error came from candidates using their calculators in degree mode rather than radian mode. Part (c) was generally done well. Some candidates chose an unsuitably large interval and some worked in degrees. Candidates who performed further iterations gained the marks provided they showed sufficient accuracy in their answers.

5. There were mixed responses to this question. Many candidates made very little progress and quite a number just carried out the differentiation in part (d). Reasonable diagrams to help with part (a) were rarely seen. Often terms were used without either a diagram or an explanation, leaving it to the examiner to interpret what the candidate was trying to do. The most successful approach was to consider two similar triangles POB and PO_2B_2 and forming sin α for each. Many were then unable to formulate the geometric sequence for the total area of the circles, so there were even fewer correct answers in the required simplified form. It was disappointing that so many attempts were dimensionally incorrect.

Part (c) proved to be difficult. Few dealt with the major arc of circle C_1. Answers to part (e) proved to be even more elusive. Many equated the derivative to zero and seemed happy to state that the least value occurred when $\cos\alpha = 4/\pi$. Some better efforts arrived at this point, realised that this had no solution and then tried to show that S was either a decreasing or an increasing function in the interval [π/6, π/4]. There were very few complete solutions to this part. It seems that even the best candidates for this paper are unaware that maxima and minima are local events firstly and only sometimes global maxima/minima.

6. Pure Mathematics P2

Most candidates were able to complete some, if not all parts of this question.

(a) Most candidates scored well, with an inability to differentiate log x being the most common cause of error. This part of the question was subject to a common misread of the function as $f(x) = 3e^x - \frac{1}{2}\ln(x-2)$. We often see candidates working without correct used of brackets, and here they saw brackets that were not present.

(b) have been straight forward but was made difficult by several candidates. Many candidates managed to produce the correct answer to (b) from a completely wrong answer to (a)!

(c) This was well done - most candidates obviously know how to use the ANS button on their calculators. Many candidates who went wrong had either made rounding errors, or they had assumed that $e^{-1} = 1$ when making their first substitution.

(d) This part was found more difficult. Candidates who did use an appropriate interval did not always give the values of the derivative correctly (for example, we assumed that an answer of 2.06… was a misread of a value in standard form). Some candidates did all the working correctly but did not draw any conclusion from it. Many chose an interval that did not include the root or that was too wide. A significant number ignored the wording of the question and continued the iteration process.

Core Mathematics

(a) Most candidates were confident differentiating both e^x and ln x. Some candidates misread f(x) and had ln(x – 2) instead of lnx -2. A number did not simplify their fraction to 1/(2x) but they were not penalised for this.

(b) They usually completed the rearrangement in part b) successfully although often needing several stages before reaching the answer on the paper. Some failed to replace the x with an alpha and a number had problems with their algebraic fractions.

(c) Candidates are very competent at obtaining values using an iteration formulae but some are not precise about the required number of decimal places.

(d) There was general familiarity with the change of sign method for determining a root but not always sufficient decimal places, and some intervals were too wide. Conclusions were sometimes missing. Some answers used f instead of f’, and others used an iterative approach despite the instruction in the question.

7. In part (a) many candidates wrote $1/2x$ as $2x^{-1}$ and differentiated to get $2x^{-2}$. Many candidates then converted $-2x^{-2}$ to $-1/2x^2$. Much fudging then went on to arrive at $x = ½$, particularly since a large number had *d(ln x/2)/dx = 2/x*. A fair number tried to show f(1/2) = 0. Part (b) Most candidates substituted in x = ½ correctly but a few did not know what to do with $\ln\frac{1}{4}$.

Most candidates knew what to do in part (c), though a few candidates did not actually evaluate f(4.905) or f(4.915) or did so incorrectly. The phrase "change of sign" was often not mentioned, but replaced with long convoluted statements. Part (d) was well answered but some candidates did miss lines out going from $\ln\frac{x}{2} = 1 - \frac{1}{2x}$ to $x = 2e^{1-\frac{1}{2x}}$

In part (e) the majority of candidates did well with some students though some lost marks because they could not use their calculator correctly or did not give their answers to the required number of decimal places.

8. Part (a) was answered well by those who used the quotient rule. Very few misquoted the formula. The product rule was more difficult to apply and led to more errors. Most used the given formula in part (b) and showed some numerical work together with a statement about a positive or negative answer leading to a minimum or maximum respectively.

The curve sketch was less well answered and candidates did not appear to be using their answer to part (b) to help them, nor to be considering the behaviour of the graph as $x \to \pm\infty$

9. Part (a) was usually well done by those who knew what a stationary value was although minor algebraic errors often spoilt essentially correct solutions. A few candidates either had no idea what a stationary value was, and began the question at part (c), or thought that they needed the second, rather than the first, derivative equal to zero. Part (b) proved the hardest part of this question and many were not able to use a correct law of logarithms to find k. Part (c) was very well done but part d(i) produced some rather uncertain work. Many candidates did not seem to realise what was expected. Nearly all candidates knew the technique required in part (d)(ii), although a few substituted 0.13 and 0.14 into f(x). The examiners do require a reason and a conclusion at the end of such a question.