1. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \le t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

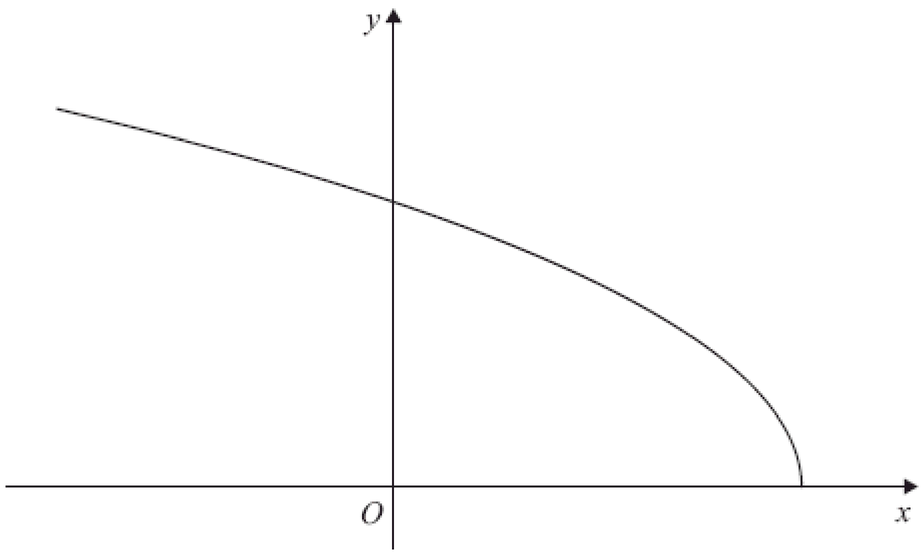
(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)
(Total 10 marks)

2.



The diagram above shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \le t \le \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \dfrac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), -k \le x \le k,$$

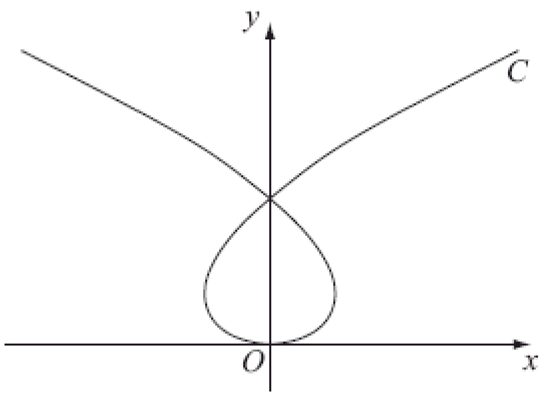
stating the value of the constant k.

(4)

(c) Write down the range of f (x).

(2)
(Total 10 marks)

3.



The curve C shown above has parametric equations

$$x = t^3 - 8t,\ y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)
(Total 12 marks)

4. A curve has parametric equations

$$x = 7\cos t - \cos 7t,\ y = 7\sin t - \sin 7t, \qquad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

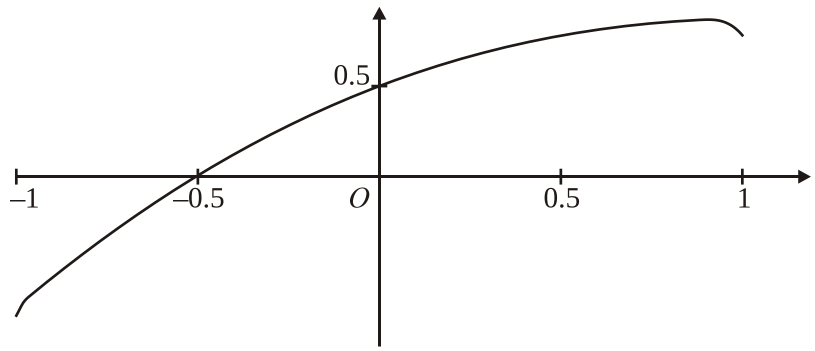
(3)

(b) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.

Give your answer in its simplest exact form.

(6)
(Total 9 marks)

5.



The curve shown in the figure above has parametric equations

$$x = \sin t,\ \ y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

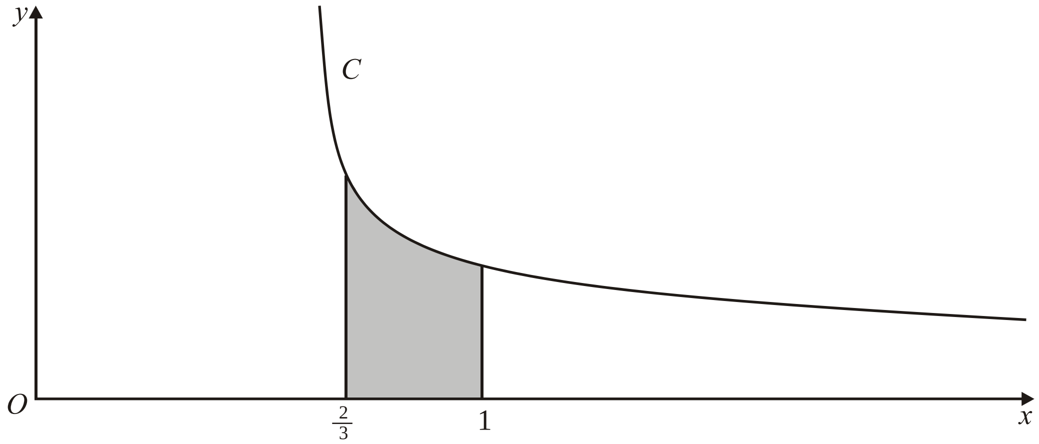
(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}, \quad -1 < x < 1$$

(3)
(Total 9 marks)

6.



The curve C has parametric equations

$$x = \frac{1}{1+t}, \quad y = \frac{1}{1-t}, \quad |t| < 1.$$

(a) Find an equation for the tangent to C at the point where $t = \frac{1}{2}$.

(7)

(b) Show that C satisfies the cartesian equation $y = \dfrac{x}{2x-1}$.

(3)

The finite region between the curve C and the x-axis, bounded by the lines with equations $x = \frac{2}{3}$ and $x = 1$, is shown shaded in the figure above.

(c) Calculate the exact value of the area of this region, giving your answer in the form $a + b \ln c$, where a, b and c are constants.

(6)
(Total 16 marks)

7. A curve has parametric equations

$$x = 2 \cot t, \quad y = 2 \sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t.

(4)

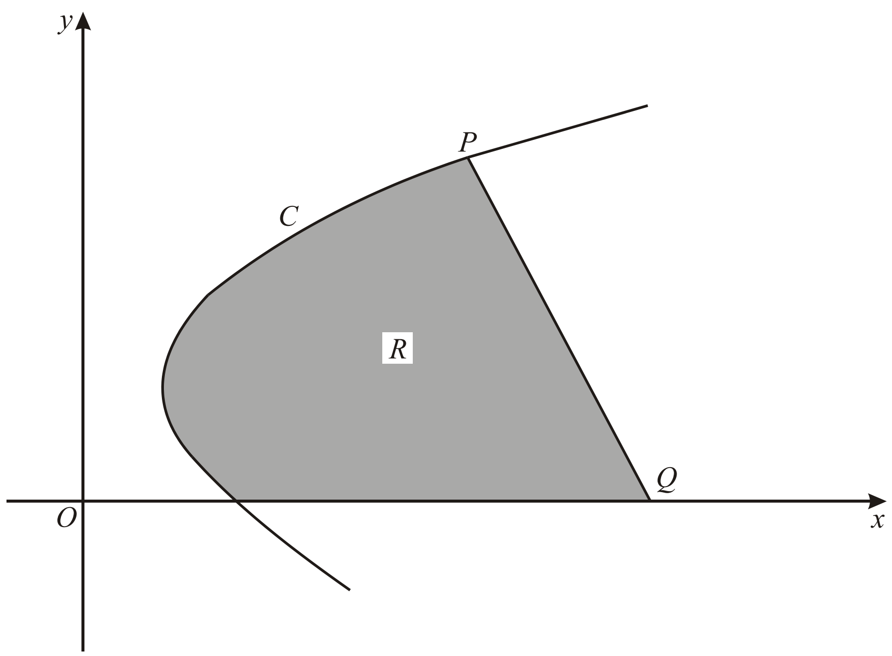
(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(c) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined.

(4)
(Total 12 marks)

8.



The diagram shows a sketch of part of the curve C with parametric equations

$$x = t^2 + 1, \qquad y = 3(1 + t).$$

The normal to C at the point $P(5, 9)$ cuts the x-axis at the point Q, as shown in the diagram.

(a) Find the x-coordinate of Q.

(6)

(b) Find the area of the finite region R bounded by C, the line PQ and the x-axis.

(9)
(Total 15 marks)

9. (a) Use the identity for $\cos(A + B)$ to prove that $\cos 2A = 2\cos^2 A - 1$.

(2)

(b) Use the substitution $x = 2\sqrt{2}\sin\theta$ to prove that

$$\int_2^{\sqrt{6}} \sqrt{(8 - x^2)}\,\mathrm{d}x = \tfrac{1}{3}(\pi + 3\sqrt{3} - 6).$$

(7)

A curve is given by the parametric equations

$$x = \sec\theta, \quad y = \ln(1 + \cos 2\theta), \quad 0 \le \theta < \frac{\pi}{2}.$$

(c) Find an equation of the tangent to the curve at the point where $\theta = \dfrac{\pi}{3}$.

(5)
(Total 14 marks)

10. The curve C has parametric equations

$$x = a\sec t, \quad y = b\tan t, \quad 0 < t < \frac{\pi}{2},$$

where a and b are positive constants.

(a) Prove that $\frac{dy}{dx} = \frac{b}{a}$ cosec t.

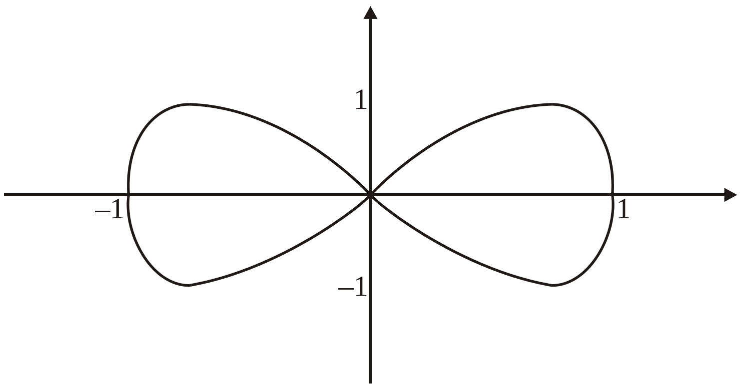
(4)

(b) Find the equation in the form $y = px + q$ of the tangent to C at the point where $t = \frac{\pi}{4}$.

(4)

(Total 8 marks)

11.



The curve shown in the diagram above has parametric equations

$$x = \cos t, \quad y = \sin 2t, \qquad 0 \le t < 2\pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t.

(3)

(b) Find the values of the parameter t at the points where $\frac{dy}{dx} = 0$.

(3)

(c) Hence give the exact values of the coordinates of the points on the curve where the tangents are parallel to the x-axis.

(2)

(d) Show that a cartesian equation for the part of the curve where $0 \leq t < \pi$ is

$$y = 2x\sqrt{(1 - x^2)}.$$

(3)

(e) Write down a cartesian equation for the part of the curve where $\pi \leq t < 2\pi$.

(1)
(Total 12 marks)

1. (a) $\frac{dx}{dt} = 2\sin t \cos t,\ \frac{dy}{dt} = 2\sec^2 t$ B1 B1

$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t \cos t}\left(= \frac{1}{\sin t \cos^3 t}\right)$ or equivalent 4

(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$ B1

$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin \frac{\pi}{3} \cos \frac{\pi}{3}} = \frac{16}{\sqrt{3}}$ A1

$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$

$y = 0 \quad \Rightarrow \quad x = \frac{3}{8}$ A1 6

2. (a) $\frac{dx}{dt} = -4\sin 2t,\ \frac{dy}{dt}\ 6\cos t$ B1, B1

$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}\left(= -\frac{3}{4\sin t}\right)$

At t = $\frac{\pi}{3}$, $m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt –0.87 A1 4

Alternatives to (a) where the parameter is eliminated

1 $y = (18 - 9x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(18 - 9x)^{-\frac{1}{2}} \times (-9)$ B1

At $t = \frac{\pi}{3}, x = \cos\frac{2\pi}{3} = -1$ B1

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$ A1 4

2 $y^2 = 18 - 9x$

$2y\frac{dy}{dx} = -9$ B1

At $t = \frac{\pi}{3}, y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$ B1

		$\frac{dy}{dx} = -\frac{9}{2\times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$		A1	4
	(b)	Use of $\cos 2t = 1 - 2\sin^2 t$			
		$\cos 2t = \frac{x}{2}, \sin t = \frac{y}{6}$			
		$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$			
		Leading to $y = \sqrt{(18-9x)} (= 3\sqrt{(2-x)})$	cao	A1	
		$-2 \le x \le 2$	$k = 2$	B1	4
	(c)	$0 \le f(x) \le 6$	either $0 \le f(x)$ or $f(x) \le 6$	B1	
			Fully correct. Accept $0 \le y \le 6$, [0, 6]	B1	2

[10]

3.	(a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	$A(7, 1)$	B1	1
	(b)	$x = t^3 - 8t$, $y = t^2$,			
		$\frac{dx}{dt} = 3t^2 - 8, \frac{dy}{dt} = 2t$			
		$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$		
			Correct $\frac{dy}{dx}$	A1	
		At A, m(**T**) = $\underline{\frac{2(-1)}{3(-1)^2 - 8}} = \underline{\frac{-2}{3-8}} = \underline{\frac{-2}{-3}} = \underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:		
		T: y – (their 1) = m_r(x – (their 7))	Finding an equation of a tangent with their point and their tangent gradient		
		or 1 = $\frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$	or finds c and uses y = (their gradient)x + "c" .	dM1	
		Hence **T**: $y = \frac{2}{5}x - \frac{9}{5}$			
		gives **T**: $\underline{2x - 5y - 9 = 0}$ **AG**		A1 cso	5

(c) $2(t^3 - 8t) - 5t^2 - 9 = 0$ — Substitution of both $x = t^3 - 8t$ and $y = t^2$ into **T**

$2t^3 - 5t^2 - 16t - 9 = 0$

$(t + 1)\{(2t^2 - 7t - 9) = 0\}$

$(t + 1)\{(t + 1)(2t - 9) = 0\}$ — A realisation that $(t + 1)$is a factor. dM1

$\{t = -1(\text{at } A)\ t = \frac{9}{2} \text{ at } B\}$ — $t = \frac{9}{2}$ A1

$x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1

Candidate uses their value of t to find either the x or y coordinate ddM1

$y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25$ or awrt 20.3 — One of either x or y correct. A1

Both x and y correct. A1 6

Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$ awrt

[12]

4. (a) $x = 7\cos t - \cos 7t,\ y = 7\sin t - \sin 7t,$

$$\frac{dx}{dt} = -7\sin t + 7\sin 7t,\ \frac{dy}{dt} = 7\cos t - 7\cos 7t$$

$$\therefore \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$$

Attempt to differentiate x **and** y with respect to t to give

$\frac{dx}{dt}$ in the form $\pm A \sin t \pm B \sin 7t$

$\frac{dy}{dt}$ in the form $\pm C \cos t \pm D \cos 7t$

Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ A1

Candidate's $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ B1ft 3

Aliter
Way 2

$x = 7\cos t - \cos 7t,\ y = 7\sin t - \sin 7t,$

$$\frac{dx}{dt} = -7\sin t + 7\sin 7t,\ \frac{dy}{dt} = 7\cos t - 7\cos 7t$$

$$\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t} = \frac{-7(-2\sin 4t\sin 3t)}{-7(2\cos 4t\sin 3t)} = \tan 4t$$

Attempt to differentiate x **and** y with respect to t to give

$\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$

$\frac{dx}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$

Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ A1

Candidate's $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ B1ft 3

(b) When $t = \frac{\pi}{6}$, $m(\mathbf{T}) = \frac{dy}{dx} = \dfrac{7\cos\frac{\pi}{6} - 7\cos\frac{7\pi}{6}}{-7\sin\frac{\pi}{6} + 7\sin\frac{7\pi}{6}}$;

$$= \underline{\frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}}} = \underline{\frac{7\sqrt{3}}{-7}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$$

Hence $m(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58

When $t = \frac{\pi}{6}$,

$$x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$$

$$y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{3} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$$

N: $y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$

N: $\underline{y = \frac{1x}{\sqrt{3}}}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$

or $4 = \frac{1}{\sqrt{3}}(4\sqrt{3}) + c \Rightarrow c = 4 - 4 = 0$

Hence **N**: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$

Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression;

to give any of the four underlined expressions oe
(must be correct solution only) A1 **cso**

Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient". A1ftoe.

The point $\underline{(4\sqrt{3},4)}$ or $\underline{(\text{awrt } 6.9,\ 4)}$ B1

Finding an equation of a normal with their point and their normal gradient or finds c by using y = (their gradient)x + "c".

Correct simplified EXACT equation of <u>normal</u>.
This is dependent on candidate using correct $(4\sqrt{3},4)$ <u>A1</u> oe 6

Aliter
Way 2

When $t = \frac{\pi}{6}$, $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \tan\frac{4\pi}{6}$;

$$= \underline{\frac{2\left(\frac{\sqrt{3}}{2}\right)(1)}{2\left(-\frac{1}{2}\right)(1)}} = \underline{-\sqrt{3}} = \underline{\text{awrt} -1.73}$$

Hence $\mathrm{m}(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58

When $t = \frac{\pi}{6}$,

$$x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$$

$$y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$$

N: $y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$

N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$

or $4 = \frac{1}{\sqrt{3}}(4\sqrt{3}) + c \Rightarrow c = 4 - 4 = 0$

Hence **N**: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$

Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression;

to give any of the four underlined expressions oe
(must be correct solution only) A1 **cso**

Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient". A1ftoe.

The point $\underline{(4\sqrt{3},4)}$ or $\underline{(\text{awrt } 6.9, 4)}$ B1

Finding an equation of a normal with their point and their normal gradient or finds c by using y = (their gradient)x + "c".

Correct simplified EXACT equation of <u>normal</u>.
This is dependent on candidate using correct $(4\sqrt{3},4)$ <u>A1</u> oe 6

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) $\to \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$.
If they write, however, **N**: $x = 4\sqrt{3}$, then they can score

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains if they write $y - 4 = 0(x - 4\sqrt{3})$ or $y = 4$.

[9]

5. (a) x = sin t y = sin(t + $\frac{\pi}{6}$)

Attempt to differentiate both x and y wrt t to give two terms in cos

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \cos t,\ \frac{\mathrm{d}y}{\mathrm{d}t} = \cos\left(t + \tfrac{\pi}{6}\right)$$ A1

Correct $\frac{\mathrm{d}x}{\mathrm{d}t}$ *and* $\frac{\mathrm{d}y}{\mathrm{d}t}$

When $t = \frac{\pi}{6}$,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$$ A1

Divides in correct way and substitutes for t to give any of the four underlined oe:
Ignore the double negative if candidate has differentiated sin $\to -$cos

when $t=\frac{\pi}{6}$, $x=\frac{1}{2}$, $y=\frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)}$ *or* $\underline{(\frac{1}{2}, awrt\ 0.87)}$

T: $\underline{y-\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}(x-\frac{1}{2})}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses y = (their gradient) x + "c". dM1

Correct EXACT equation of tangent oe. A1 oe

or $\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}\left(\frac{1}{2}\right)+c \Rightarrow c=\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{6}=\frac{\sqrt{3}}{3}$

or T: $\left\lfloor \underline{y=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}} \right\rfloor$ 6

(b) $y = \sin(t + \frac{\pi}{6}) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$

Use of compound angle formula for sine.

Nb: $\sin^2 t + \cos^2 t = 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

$\therefore x = \sin t$ gives $\cos t = \sqrt{(1-x^2)}$

Use of trig identity to find cos t in terms of x or $\cos^2 t$ in terms of x.

$\therefore y=\frac{\sqrt{3}}{2}\sin t+\frac{1}{2}\cos t$

gives $y=\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{(1-x^2)}$ AG A1 cso 3

Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, *cost and sin* $\frac{\pi}{6}$ *to give y in terms of x.*

[9]

Aliter Way 2

(a) $x = \sin t$ $y = \sin(t + \frac{\pi}{6}) = \sin t + \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$

(Do not give this for part (b))

Attempt to differentiate x and y wrt t to give $\frac{dx}{dy}$ *in terms of cos and* $\frac{dy}{dt}$ *in the form* $\pm a \cos t \pm b \sin t$

$\frac{dx}{dt} = \cos t$; $\frac{dy}{dt} = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$ A1

Correct $\frac{dx}{dt}$ *and* $\frac{dy}{dt}$

When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ A1

Divides in correct way and substitutes for t to give any of the four underlined oe

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses y = (their gradient)x + "c". dM1

Correct EXACT equation of <u>*tangent*</u> *oe.* A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + \text{c} \Rightarrow \text{c} = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$

Aliter Way 3

(a) $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$

Attempt to differentiate two terms using the chain rule for the second term.

Correct $\frac{dy}{dx}$ A1

$\frac{dy}{dx} = \underline{\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(1-(0.5)^2\right)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}}$ A1

Correct substitution of x = $\frac{1}{2}$ *into a correct* $\frac{dy}{dx}$

When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ B1

The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ *or* $\underline{\left(\frac{1}{2}, awrt\ 0.87\right)}$

T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$

Finding an equation of a tangent with their point and their tangent gradient or finds c and uses y = (their gradient) x + "c" dM1

Correct <u>EXACT</u> equation of <u>tangent</u> oe. A1 oe

or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

or **T**: $\left\lfloor \underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}} \right\rfloor$ 6

Aliter Way 2

(b) $x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$

Substitutes x = $\sin t$ *into the equation give in y.*

Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$

Cost = $\sqrt{(1-\sin^2 t)}$

Use of trig identity to deduce that cos t = $\sqrt{(1-\sin^2 t)}$

gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$

Hence $y = \sin t\cos\frac{\pi}{6} + \cos t\sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$ A1 cso 3

Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$

6. (a) $\frac{dx}{dt} = -\frac{1}{(1+t)^2}$ and $\frac{dy}{dt} = \frac{1}{(1-t)^2}$ B1, B1

$\therefore \frac{dy}{dx} = \frac{-(1+t)^2}{(1-t)^2}$ and at $t = ½$, gradient is -9 A1cao

requires their dy/dt / their dx/dt and substitution of t.

At the point of contact $x = \frac{2}{3}$ and $y = 2$ B1

Equation is $y - 2 = -9\left(x - \frac{2}{3}\right)$ A1 7

(b) **Either** obtain t in terms of x and y i,e, $t = \frac{1}{x} - 1$ or $t = 1 - \frac{1}{x}$ (or both)

Then substitute into other expression $y = f(x)$ or $x = g(y)$ and rearrange

(or put $\frac{1}{x} - 1 = 1 - \frac{1}{y}$ and rearrange)

To obtain $y = \frac{x}{2x-1}$ (*) a1 3

Or Substitute into $\frac{x}{2x-1} = \frac{\frac{1}{(1+t)}}{\frac{2}{1+t} - 1}$

$= \frac{1}{2-(1+t)} = \frac{1}{1-t}$ A1

$= y$ (*) 3

(c) Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,dx$ B1

$= \int \frac{u+1}{2u}\frac{du}{2} = \frac{1}{4}\int 1 + \frac{1}{u}\,du$

putting into a form to integrate

$= \left[\frac{1}{4}u + \frac{1}{4}\ln u\right]_{\frac{1}{3}}^{1}$ A1

$= \frac{1}{4} - \left(\frac{1}{12} + \frac{1}{4}\ln\frac{1}{3}\right)$

$= \frac{1}{6} + \frac{1}{4}\ln 3$ or any correct equivalent. A1 6

[16]

Or Area = $\displaystyle\int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,\mathrm{d}x$ B1

$= \displaystyle\int \frac{1}{2} + \frac{\frac{1}{2}}{2x-1}\,\mathrm{d}x$

putting into a form to integrate

$= \left[\frac{1}{2}x + \frac{1}{4}\ln(2x-1)\right]_{\frac{2}{3}}^{1}$ A1

$= \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\ln\frac{1}{3} = \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$ dM1 A1 6

Or Area = $\displaystyle\int \frac{1}{1-t}\frac{-1}{(1+t)^2}\,\mathrm{d}t$ B1

$= \displaystyle\int \frac{A}{(1-t)} + \frac{B}{(1+t)} + \frac{C}{(1+t)^2}\,\mathrm{d}t$

putting into a form to integrate

$= \left[\frac{1}{4}\ln(1-t) - \frac{1}{4}\ln(1+t) + \frac{1}{2}(1+t)^{-1}\right]$ a1ft

= Using limits 0 and ½ and subtracting (either way round) dM1

$= \frac{1}{6} + \frac{1}{4}$ ln3 or any correct equivalent. A1 6

Or Area = $\displaystyle\int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,\mathrm{d}x$ then use parts B1

$= \frac{1}{2}x\ln(2x-1) - \displaystyle\int_{\frac{2}{3}}^{1} \frac{1}{2}(2x-1)\,\mathrm{d}x$

$= \frac{1}{2}x\ln(2x-1) - \left[\frac{1}{4}(2x-1)\ln(2x-1) - \frac{1}{2}x\right]$ M1A1

$= \frac{1}{2} - \left(\frac{1}{3}\ln\frac{1}{3} - \frac{1}{12}\ln\frac{1}{3} + \frac{1}{3}\right)$ DM1

$= \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$ A1 6

7. (a) $\frac{\mathrm{d}x}{\mathrm{d}t} = -2\operatorname{cosec}^2 t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 4\sin t\cos t$ both A1

$\frac{dy}{dx} = \frac{-2\sin t \cos t}{cosec^2 t}$ (= $-2\sin^3 t \cos t$) A1 4

(b) At $t = \frac{\pi}{4}$, $x = 2$, $y = 1$ B1

both x and y

Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$

Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$ A1 4

Accept x + 2y = 4 or any correct equivalent

(c) Uses $1 + \cot^2 t = \text{cosec}^2 t$, or equivalent, to eliminate t

$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ A1

correctly eliminates t

$y = \frac{8}{4 + x^2}$ cao A1

The domain ir $x \ldots 0$ B1 4

Alternative for (c):

$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$

$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$ A1

Leading to $y = \frac{8}{4 + x^2}$ A1

[12]

8. (a) $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{3}{2t}$

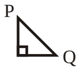
Gradient of normal is $-\frac{2t}{3}$

At P $t = 2$ B1

$\therefore$ Gradient of normal @ P is $-\frac{4}{3}$ A1

Equation of normal @ P is $y - 9 = -\frac{4}{3}(x - 5)$

Q is where $y = 0$ $\therefore$ $x = \frac{27}{4} + 5 = \underline{\frac{47}{4}}$ (o.e.) A1 6

	(b)	Curved area = $\int y\,\mathrm{d}x = \int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$		
		$= \int 3(1 + b).2t\ \mathrm{d}t$	A1	
		$= [3t^2 + 2t^3]$	M1A1	
		Curve cuts x-axis when $t = -1$	B1	
		Curved area = $[3t^2 + 2t^3]_{-1}^{2} = (12 + 16) - (3 - 2)\ (= 27)$		
		Area of  triangle = $\frac{1}{2}((\mathrm{a}) - 5) \times 9\ (= 30.375)$		
		Total area of R = curved area + Δ		
		= 57.375 or AWRT <u>57.4</u>	A1	9
				[15]
9.	(a)	$\cos(A + A) = \cos^2 A - \sin^2 A$		
		$= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$	A1	2
	(b)	$[x = 2,\ \theta = \frac{\pi}{4};\ x = \sqrt{6},\ \theta = \frac{\pi}{3}]$	B1	
		$x = 2\sqrt{2}\sin\theta,\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sqrt{2}\cos\theta$	B1	
		$\int\sqrt{8 - x^2}\,\mathrm{d}x = \int 2\sqrt{2}\cos\theta 2\sqrt{2}\cos\theta\mathrm{d}\theta = \int 8\cos^2\theta\mathrm{d}\theta$	A1	
		Using $\cos 2\theta = 2\cos^2\theta - 1$ to give $\int 4(1 + \cos 2\theta)\mathrm{d}\theta$	dM1	
		$= 4\theta + 2\sin 2\theta$	A1 ft	
		Substituting limits to give $\frac{1}{3}\pi + \sqrt{3} - 2$ or given result	A1	7
	(c)	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{-2\sin 2\theta}{1 + \cos 2\theta}$	B1	
		Using the chain rule, with $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \sec\theta\tan\theta$ to give $\frac{\mathrm{d}y}{\mathrm{d}x}(= -2\cos\theta)$		
		Gradient at the point where $\theta = \frac{\pi}{3}$ is –1.	A1 ft	
		Equation of tangent is $y + \ln 2 = -(x - 2)$ (o.a.e.)	A1	5
				[12]
10.	(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = a\sec t\tan t$	B1	
		$\frac{\mathrm{d}y}{\mathrm{d}t} = b\sec^2 t$	B1	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{b\sec^2 t}{a\sec t\tan t} = \frac{b}{a}\operatorname{cosec} t$	A1	4

	(b)	At $t = \dfrac{\pi}{4}$ $\quad x = a\sqrt{2},\ y = b;\ \dfrac{dy}{dx} = \dfrac{b\sqrt{2}}{a}$	B1B1	
		$(y - b) = \dfrac{b\sqrt{2}}{a}\left(x - a\sqrt{2}\right)$ $y = \dfrac{b\sqrt{2}x}{a} - b$	A1	4
				[8]
11.	(a)	$\dfrac{dx}{dt} = -\sin t,\quad \dfrac{dy}{dt} = 2\cos 2t \quad \therefore \dfrac{dy}{dx} = \dfrac{2\cos 2t}{-\sin t}$	A1 A1	3
	(b)	$2\cos 2t = 0 \quad \therefore 2t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2}$ So $t = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$	A1 A1	3
	(c)	$\left(\dfrac{1}{\sqrt{2}}, 1)\right)\ \left(\dfrac{1}{\sqrt{2}}, -1)\right)\ \left(-\dfrac{1}{\sqrt{2}}, 1)\right)\ \left(-\dfrac{1}{\sqrt{2}}, -1)\right)$	A1	2
	(d)	$y = 2\sin t\cos t$ $= 2\sqrt{1-\cos^2 t}\cos t = 2x\sqrt{1-x^2}$	A1	3
	(e)	$y = -2x\sqrt{1-x^2}$	B1	1
				[12]

1. The majority of candidates knew how to tackle this question and solutions gaining all the method marks were common. However there were many errors of detail and only about 32% of the candidates gained full marks. In part (a), many candidates had difficult in differentiating $\sin^2 t$ and $2\tan t$. $2\tan t$ was more often differentiated correctly, possibly because the differential of $\tan t$ is given in the formula book, although 2 ln sec$t$ or ln sec^2 t were often seen. Many could not differentiate $\sin^2 t$ correctly. $\cos^2 t$, 2 cos t and 2 sin t were all common. Nearly all candidates knew they had to divide $\frac{dy}{dt}$ by $\frac{dx}{dt}$, although there was some confusion in notation, with candidates mixing up their xs and ts. The majority knew how to approach part (b), finding the linear equation of the tangent to the curve at $\left(\frac{3}{4}, 2\sqrt{3}\right)$, putting $y = 0$ and solving for x. Some candidates used $y = 0$ prematurely and found the tangent to the curve at $\left(\frac{3}{4}, 0\right)$ rather than at $\left(\frac{3}{4}, 2\sqrt{3}\right)$.

2. Nearly all candidates knew the method for solving part (a), although there were many errors in differentiating trig functions. In particular $\frac{d}{dt}$ (2cos2t) was often incorrect. It was clear from both this question and question 2 that, for many, the calculus of trig functions was an area of weakness. Nearly all candidates were able to obtain an exact answer in surd form. In part (b), the majority of candidates were able to eliminate t but, in manipulating trigonometric identities, many errors, particularly with signs, were seen. The answer was given in a variety of forms and all exact equivalent answers to that printed in the mark scheme were accepted. The value of k was often omitted and it is possible that some simply overlooked this. Domain and range remains an unpopular topic and many did not attempt part (c). In this case, inspection of the printed figure gives the lower limit and was intended to give candidates a lead to identifying the upper limit.

3. Part (a) was answered correctly by almost all candidates. In part (b), many candidates correctly applied the method of finding a tangent by using parametric differentiation to give the answer in the correct form. Few candidates tried to eliminate t to find a Cartesian equation for C, but these candidates were usually not able to find the correct gradient at A.

In part (c), fully correct solutions were much less frequently seen. A significant number of candidates were able to obtain an equation in one variable to score the first method mark, but were then unsure about how to proceed. Successful candidates mostly formed an equation in t, used the fact that $t + 1$ was a factor and applied the factor theorem in order for them to find t at the point B. They then substituted this t into the parametric equations to find the coordinates of B. Those candidates who initially formed an equation in y only went no further. A common misconception in part (c), was for candidates to believe that the gradient at the point B would be the same as the gradient at the point A and a significant minority of candidates attempted to solve $\frac{2t}{3t^2 - 8} = \frac{2}{5}$ to find t at the point B.

4. In part (a), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of t. Some candidates erroneously believed that differentiation of a sine function produced a negative cosine function and the differentiation of a cosine function produced a positive sine function. Other candidates incorrectly differentiated cos $7t$ to give either $-\frac{1}{7}\sin 7\,t$ or $-\sin 7t$ and also incorrectly differentiated sin $7t$ to give either $\frac{1}{7}\cos 7t$ or $\cos 7t$.

In part (b), many candidates were able to substitute $t = \frac{\pi}{6}$ into their gradient expression to give $-\sqrt{3}$, but it was not uncommon to see some candidates who made errors when simplifying their substituted expression. The majority of candidates were able to find the point ($4\sqrt{3}$, 4). Some candidates, however, incorrectly evaluated cos ($\frac{7\pi}{6}$) and sin ($\frac{7\pi}{6}$) as $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$ respectively and found the incorrect point ($3\sqrt{3}$, 3). Some candidates failed to use the gradient of the tangent to find the gradient of the normal and instead found the equation of the tangent, and so lost valuable marks as a result. It was pleasing to see that a significant number of candidates were able to express the equation of the normal in its simplest exact form.

5. Part (a) was surprisingly well done by candidates with part (b) providing more of a challenge even for some candidates who had produced a perfect solution in part (a).
In part (a), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of t, although some candidates erroneously believed that differentiation of a sine function produced a negative cosine function. Other mistakes included a few candidates who either cancelled out "cos" in their gradient expression to give $\frac{t+\frac{\pi}{6}}{t}$ or substituted t = $\frac{\pi}{6}$ into their x and y expressions before proceeding to differentiate each with respect to t. Other candidates made life more difficult for themselves by expanding the y expression using the compound angle formula, giving them more work, but for the same credit. Many candidates were able to substitute t=$\frac{\pi}{6}$ into their gradient expression to give $\frac{1}{\sqrt{3}}$, but it was not uncommon to see some candidates who simplified $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ incorrectly to give $\sqrt{3}$ The majority of candidates wrote down the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and understood how to find the equation of the tangent using this point and their tangent gradient.
Whilst some candidates omitted part (b) altogether, most realised they needed to use the compound angle formula, though it was common to see that some candidates who believed that sin (t + $\frac{\pi}{6}$)could be rewritten as ' sin$t$ + sin$\frac{\pi}{6}$ '. Many candidates do not appreciate that a proof requires evidence, as was required in establishing that cos t = $\sqrt{1-x^2}$, and so lost the final two marks. There were, however, a significant number of candidates who successfully obtained the required Cartesian equation.

6. (a) A number of candidates lost marks in this question. Some confused differentiation with integration and obtained a logarithm, others made sign slips differentiating y, and a number who obtained the correct gradient failed to continue to find the equation of the tangent using equations of a straight line.

(b) There was a lack of understanding of *proof* with a number of candidates merely substituting in values. Better candidates were able to begin correctly but some did not realise that if the answer is given it is necessary to show more working.

(c) Very few got this correct. There was a tendency to use parts and to be unable to deal with the integral of ln(2x–1). The most successful methods involved dividing out, or substituting for (2x-1). Those who tried a parametric approach rarely recognised the need for partial fractions.

7. This proved a testing question and few could find both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correctly. A common error was to integrate x, giving $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\ln(\sin t)$. Most knew, however, how to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ from $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ and were able to pick up marks here and in part (b). In part (b), the method for finding the equation of the tangent was well understood. Part (c) proved very demanding and only a minority of candidates were able to use one of the trigonometric forms of Pythagoras to eliminate t and manipulate the resulting equation to obtain an answer in the required form. Few even attempted the domain and the fully correct answer, x□ 0, was very rarely seen

8. Many good attempts at part (a) were seen by those who appreciated $t = 2$ at P, or by those using the Cartesian equation of the curve. Algebraic errors due to careless writing led to a loss of accuracy throughout the question, most commonly

$$\frac{3}{2t} \to \frac{3}{2}t \to \frac{3t}{2}$$

Part (b) undoubtedly caused candidates extreme difficulty in deciding which section of the shaded area R was involved with integration. The majority set up some indefinite integration and carried this out well. Only the very able sorted out the limits satisfactorily. Most also evaluated the area of either a triangle or a trapezium and combined, in some way, this with their integrand, demonstrating to examiners their overall understanding of this situation.

9. Most candidates understood the requirements of the proof of the double angle formula in part (a). Part (b) proved to be discriminating, but a large number of candidates produced good solutions, where they changed the variables and the limits and used the appropriate double angle formula to perform the integration. Some difficulties were experienced differentiating the log function in part (c), but again there were a large number of correct solutions. A few candidates eliminated the parameter and found the cartesian equation of the curve before differentiation.

10. No Report available for this question.

11. No Report available for this question.