1. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when t = 22 920.

(2)

(d) In the space provided on page 13, sketch the graph of R against t.

(2)
(Total 9 marks)

2. (i) The curve C has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = \left(1 + e^{2x}\right)^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)
(Total 11 marks)

3. The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 2x + \ln 2, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \rightarrow \mathrm{e}^{2x}, \qquad x \in \mathbb{R}$$

(a) Prove that the composite function gf is

$$\mathrm{gf} : x \rightarrow 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}$$

(4)

(b) In the space provided below, sketch the curve with equation y = gf(x), and show the coordinates of the point where the curve cuts the y-axis.

(1)

(c) Write down the range of gf.

(1)

(d) Find the value of x for which $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)
(Total 10 marks)

4. Given that $y = \log_a x$, $x > 0$, where a is a positive constant,

(a) (i) express x in terms of a and y,

(1)

(ii) deduce that $\ln x = y \ln a$.

(1)

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x \ln a}$.

(2)

The curve C has equation $y = \log_{10} x$, $x > 0$. The point A on C has x-coordinate 10. Using the result in part (b),

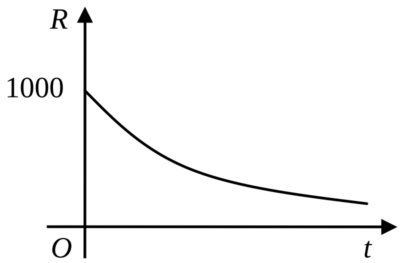
(c) find an equation for the tangent to C at A.

(4)

The tangent to C at A crosses the x-axis at the point B.

(d) Find the exact x-coordinate of B.

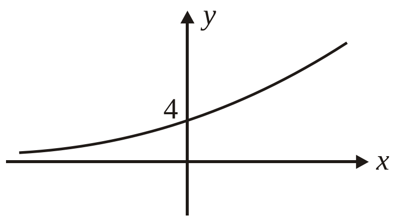
(2)
(Total 10 marks)

1.	(a)	1000		B1	1
	(b)	$1000e^{-5730c} = 500$			
		$e^{-5730c} = \frac{1}{2}$		A1	
		$-5730c = \ln\frac{1}{2}$			
		$c = 0.000121$	cao	A1	4
	(c)	$R = 1000e^{-22920c} = 62.5$	Accept 62-63	A1	2
	(d)				
		Shape		B1	
		1000		B1	2

[9]

2.	(i)	$\frac{dy}{dx} = \frac{(9+x^2)-x(2x)}{(9+x^2)^2} \left(= \frac{9-x^2}{(9+x^2)^2} \right)$	M1A1	
		$\frac{dy}{dx} = 0 \Rightarrow 9 - x^2 = 0 \Rightarrow x = \pm 3$	M1A1	
		$\left(3, \frac{1}{6}\right), \left(-3, -\frac{1}{6}\right)$ Final two A marks depend on second M only	A1, A1	6
	(ii)	$\frac{dy}{dx} = \frac{3}{2}\left(1+e^{2x}\right)^{\frac{1}{2}} \times 2e^{2x}$	M1A1A1	
		$x = \frac{1}{2}\ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2}(1+e^{\ln 3})^{\frac{1}{2}} \times 2e^{\ln 3} = 3 \times 4^{\frac{1}{2}} \times 3 = 18$	M1A1	5

[11]

3.	(a)	$gf(x) = e^{2(2x + \ln 2)}$		
		$= e^{4x}e^{2\ln 2}$		
		$= e^{4x}e^{\ln 4}$		
		$= 4e^{4x}$ **AG**	A1	4
	(b)		B1 shape & (0, 4)	1
	(c)	$gf(x) > 0$	B1	1
	(d)	$\frac{d}{dx} gf(x) = 16e^{4x}$		
		$e^{4x} = \frac{3}{16}$	attempt to solve	
		$4x = \ln \frac{3}{16}$	A1	
		$x = -0.418$	A1	4

[10]

4. (a) (i) $\underline{x = a^y}$ B1 1

(ii) In both sides of (i) i.e ln x = ln a^y or (y =) $\log_a x = \dfrac{\ln x}{\ln a}$

$= \underline{y \ln a}$ * $\Rightarrow y \ln a = \ln x$ B1$_{cso}$ 1

B1 $x = e^{y\ln a}$ is BO

B1 Must see ln a^y or use of change of base formula.

(b) $y = \dfrac{1}{\ln a} \bullet \ln x, \Rightarrow \dfrac{dy}{dx}, = \underline{\dfrac{1}{\ln a} \times \dfrac{1}{x}}$ * A1$_{cso}$ 2

ALT. $\left[\text{or} \quad \dfrac{1}{x} = \dfrac{dy}{dx} \cdot \ln a \quad , \Rightarrow \quad \underline{\dfrac{dy}{dx} = \dfrac{1}{x \ln a}} \quad * \right]$

A1$_{cso}$ needs some correct attempt at differentiating.

(c) $\log_{10} 10 = 1 \Rightarrow$ A is (10, $\underline{1}$) $y_A = 1$ B1

from(b) $\underline{m = \dfrac{1}{10 \ln a} \text{ or } \dfrac{1}{10 \ln 10}}$ or 0.043 (or better) B1

equ of target $y - 1 = m(x - 10)$

i.e $\underline{y - 1 = \dfrac{1}{10 \ln 10}(x - 10) \text{ or } y = \dfrac{1}{10 \ln 10} x + 1 - \dfrac{1}{\ln 10}}$ (o.e) A1 4

B1 Allow either

ft their y_A and m

(d) $y = 0$ *in* (c) $\Rightarrow 0 = \dfrac{x}{10 \ln 10} + 1 - \dfrac{1}{\ln 10} \Rightarrow x, = 10 \ln 10 \left(\dfrac{1}{\ln 10} - 1 \right)$

$\underline{x = 10 - 10 \ln 10}$ or $\underline{10(1 - \ln 10)}$ or $\underline{10 \ln 10 (\dfrac{1}{\ln 10} - 1)}$ A1 2

Attempt to solve correct equation. Allow if a not = 10.

[10]

1. This question was well answered by a high proportion of the candidates and completely correct solutions to parts (a), (b) and (c) were common. Virtually all candidates gained the mark in part (a). A few thought the answer was 1000e.

Those who failed to get the correct answer in part (b) often substituted $R = \frac{1}{2}$ instead of $R = 500$. Logs were usually taken correctly but it was perhaps disappointing that a not insignificant number of candidates lost the final mark by failing to give the answer correctly to 3 significant figures. In part (c), most candidates scored the method mark even if they had struggled with part (b). Very few who had a correct value of c failed to go on to gain the second mark.

The graph in part (d) was generally less well done. A common error was to draw a curve of the type $y = \frac{1}{x}$ and this lost both marks. Occasionally candidates had the curve meeting the positive x-axis rather than approaching it asymptotically.

2. The techniques required to answer this question were well understood and fully correct answers to both parts of the question were common. In part (a), most used the quotient rule correctly and obtained $\frac{9-x^2}{(9+x^2)^2} = 0$. The solution of this equation caused some difficulty. Some gave up at this point but it was not uncommon to see candidates proceeding from this correct equation to $9 - x^2 = (9 + x^2)^2$ and it was possible to waste much time solving this. Those who obtained $9 - x^2 = 0$ usually completed the question correctly although some missed the solution $\left(-3, -\frac{1}{6}\right)$. There are candidates who attempt this type of question using the product rule. This is, of course, mathematically sound but it cannot be recommended for the majority of candidates. Negative indices are found difficult and, in this case, obtaining the equation $(9 + x^2)^{-1} - 2x^2 (9 + x^2)^{-2} = 0$ and solving it proved beyond all but the ablest. Part (b) was well done. The commonest error seen was $\frac{dy}{dx} = \frac{3}{2}(1+e^{2x}) \times e^{2x}$, which usually derived from the error $\frac{d}{dx}(e^{2x}) = e^{2x}$. Most could carry out the substitution and evaluation and the correct answer 18 was frequently seen.

3. Pure Mathematics P2

This proved to be the most difficult question on the paper, not necessarily because the candidates could not attempt it, but mainly due to the errors made.

A "given answer" is often an invitation to bluff that you have demonstrated more than you actually have, and that was certainly the case here. Most candidates started with the correct combination of the two functions, but few demonstrated the splitting of the index to give the product of two terms, and even fewer showed a full explanation for $e^{2\ln 2} = e^{\ln 4} = 4$.. Candidates need to appreciate the importance of setting out all stages of their working in this situation.

There were several good sketches, but there were sketches of the incorrect shape, sketches that crossed the x axis, sketches passing through (0,1), and a number where the horizontal asymptote appeared to be noticeably above $y = 0$.

Candidates' descriptions of the range of gf were often not consistent with their sketches. Incorrect answers often included 0, or the full set of real numbers. A maximum or minimum value of 4 was also a common alternative.

Most candidates demonstrated little or no understanding of how to differentiate $4e^{4x}$. In many instances the answer was $4e^{4x}$, but $4xe^{4x}$ and 4^{e3x} were also seen. Despite errors in the differentiation, many candidates reached an expression of the form $e^p = q$, but were not able to go on to use logarithms correctly to attempt to solve their equation.

Core Mathematics

Part (a) asked for a proof of a straightforward result and, with four marks available, it was expected that each step of the proof was clearly demonstrated. The examiners wanted to see that the operations were performed in the correct order, that the index law was applied to the exponential function (a step such as $e^{4x+2\ln x} = e^{4x} e^{2\ln x}$ is sufficient for this) and that a law of logarithms was used. This step could be shown by writing 2ln 2 = ln 4 or $2\ln 2 = 2^2$ or. The examiners did not require specific reference to a law of logarithms. Many gained only one or two of the marks available for part(a). The sketch was well done and more candidates seemed to be able to state the appropriate range than has been the case in some previous examinations. The differentiation at the in part (d) proved difficult for many and $\frac{d}{dx}\left(4e^{4x}\right) = 4e^{4x}, 8e^{4x}$ and $16e^{3x}$ were all seen from time to time. Apart from this, the method of solution of the equation was well understood.

4. Part (a) (i) was usually answered well but the provision of the answers in the next two parts meant that a number of candidates failed to score full marks either through failing to show sufficient working, or by the inclusion of an incorrect step or statement.

The most successful approach to part (b) started from $y = \dfrac{\ln x}{\ln a}$ but there was sometimes poor use of logs such as $\dfrac{\ln x}{\ln a} = \ln x - \ln a = \ln\left(\dfrac{x}{a}\right)$; incorrect notation such as $\dfrac{dy}{dx}\ln x = \dfrac{1}{x}$,or errors in differentiation when $\dfrac{1}{a}$ appeared.

In part (c) the process for finding an equation for the tangent was usually well known but some candidates did not appreciate that the gradient should be a constant and gave a non-linear equation for their tangent. Some confused ln10 with $\log_{10}10$. Many candidates had problems working exactly in the final part and others ignored this instruction and gave an answer of –13.02.