

Calculus Questions

2 A curve is defined by the parametric equations

$$x = 3 - 4t \quad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of t . (4 marks)

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4 marks)

(c) Verify that the cartesian equation of the curve can be written as

$$(x - 3)(y - 1) + 8 = 0 \quad (3 \text{ marks})$$

6 (a) Express $\cos 2x$ in the form $a \cos^2 x + b$, where a and b are constants. (2 marks)

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where a is an integer. (5 marks)

8 (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x - 6)^{\frac{1}{2}}$$

to find t in terms of x , given that $x = 70$ when $t = 0$. (6 marks)

(b) Liquid fuel is stored in a tank. At time t minutes, the depth of fuel in the tank is x cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x - 6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. (1 mark)

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. (2 marks)

5 A curve is defined by the equation

$$y^2 - xy + 3x^2 - 5 = 0$$

- (a) Find the y -coordinates of the two points on the curve where $x = 1$. (3 marks)
- (b) (i) Show that $\frac{dy}{dx} = \frac{y - 6x}{2y - x}$. (6 marks)
- (ii) Find the gradient of the curve at each of the points where $x = 1$. (2 marks)
- (iii) Show that, at the two stationary points on the curve, $33x^2 - 5 = 0$. (3 marks)
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7 Solve the differential equation

$$\frac{dy}{dx} = 6xy^2$$

given that $y = 1$ when $x = 2$. Give your answer in the form $y = f(x)$. (6 marks)

1 A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

- (a) (i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. (2 marks)
- (ii) Hence find $\frac{dy}{dx}$ in terms of t . (2 marks)
- (b) Find an equation of the normal to the curve at the point where $t = 1$. (4 marks)
- (c) Find a cartesian equation of the curve. (3 marks)
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- 8 (a) (i) Solve the differential equation $\frac{dy}{dt} = y \sin t$ to obtain y in terms of t . (4 marks)
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5 The point $P(1, a)$, where $a > 0$, lies on the curve $y + 4x = 5x^2y^2$.

- (a) Show that $a = 1$. (2 marks)
- (b) Find the gradient of the curve at P . (7 marks)
- (c) Find an equation of the tangent to the curve at P . (1 mark)

6 A curve is given by the parametric equations

$$x = \cos \theta \quad y = \sin 2\theta$$

(a) (i) Find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$. (2 marks)

(ii) Find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$. (2 marks)

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

8 (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1 + 2y}}{x^2}$$

given that $y = 4$ when $x = 1$. (6 marks)

(b) Show that the solution can be written as $y = \frac{1}{2} \left(15 - \frac{8}{x} + \frac{1}{x^2} \right)$. (2 marks)

Calculus Answers

2(a)	$\frac{dy}{dt} = \frac{-2}{t^2} \quad \frac{dx}{dt} = -4$ $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{1}{\frac{dx}{dt}} = \frac{1}{2t^2}$	M1A1 m1 A1F	4	Use chain rule Follow on use of chain rule (if $f(t)$) Or eliminate t : M1 $y=f(x)$ attempt to differentiate M1A1 chain rule A1F reintroduce t follow on gradient (possibly used later)
(b)	$t = 2 \quad m_T = \frac{1}{8}$ $x = -5 \quad y = 2$ $y - 2 = \frac{1}{8}(x + 5)$ $x - 8y + 21 = 0$	B1F B1 M1 A1F	4	Their $(x, y), m$ Ft on (x, y) and m
(c)	$x - 3 = -4t \quad y - 1 = \frac{2}{t}$ $(x - 3)(y - 1) = -4t \times \frac{2}{t} = (-8)$	M1 M1 A1	3	PI Attempt to eliminate t AG convincingly obtained
Total			11	

6(a)	$\cos 2x = 2 \cos^2 x - 1$	B1B1	2	
(b)	$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$ $\frac{1}{2} \int_0^{\frac{\pi}{2}} \cos 2x + 1 \, dx = \left[\frac{1}{4} \sin 2x + \frac{x}{2} \right]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{4}$	M1 A1 A1 M1A1F	5	Attempt to express $\cos^2 x$ in terms of $\cos 2x$ Use limits. Ft on integer a .
Total			7	

8(a)	$\int \frac{dx}{\sqrt{x-6}} = \int -2dt$ $2\sqrt{x-6} = -2t + c$ $t=0 \quad x=70 \Rightarrow c=16$ $t = 8 - \sqrt{x-6}$	M1 A1A1 m1A1F		Attempt to separate and integrate c on either side Follow on c from sensible attempt at integrals ($\sqrt{\quad}$ not \ln)
		A1	6	CAO (or AEF)
(b)(i)	The liquid level stops falling/flowing/ at minimum depth $x=22 \quad t=8-\sqrt{22-6}$	B1 M1	1	Use $x=22$ in their equation provided there is a c Or start again using limits M1 $2\sqrt{64}-2\sqrt{16} = \pm 2t$, A1 $t=4$
	$t=4$	A1	2	CAO
Total			9	

5(a)	$x=1 \quad y^2 - y + 3 - 5 = 0$ $(y-2)(y+1) = 0$ $y=2 \quad y=-1$	M1 M1 A1	3	Attempt to solve quadratic equation with $x=1$
(b)(i)	$2y \frac{dy}{dx} - x \frac{dy}{dx} - y + 6x = 0$ $6x - y + (2y-x) \frac{dy}{dx} = 0$ Alternative $\frac{dy}{dx}(y-x)^2 = (y-x)(0-6x)$ $-(5-3x^2) \left(\frac{dy}{dx} - 1 \right)$ $\frac{dy}{dx} [(y+x)^2 + (5-3x^2)] = (y-x)(-6x)$ $+(5-3x^2)$ Given answer	B1B1 B1 M1A1 A1 (B1) (B1) (M1) (A1) (A1) (A1)	6	+6x; $-5 \rightarrow 0$ Chain rule Product rule (M1 two terms) Factorise and obtain answer given $5 \rightarrow 0$ $-6x$ Recognisable attempt at quotient rule Completely correct OE Factorise out $\frac{dy}{dx}$ Correct answer from correct working Be convinced
(ii)	$(1, 2) \quad \frac{dy}{dx} = -\frac{4}{3}$ $(1, -1) \quad \frac{dy}{dx} = \frac{7}{3}$	M1 A1F	2	Substitute $x=1$ and one y value from (a) Both; follow on candidates y s OE $-\frac{7}{-3}$; 3SF
(iii)	$y - 6x = 0$ $(6x)^2 - x \times 6x + 3x^2 - 5 = 0$ $36x^2 - 6x^2 + 3x^2 - 5 = 0$ $33x^2 - 5 = 0$	B1 M1 A1	3	AG convincingly obtained
Total			14	

7	$\int \frac{dy}{y^2} = \int 6x \, dx$	M1		Attempt to separate Either dx or dy in right place
	$-\frac{1}{y} = 3x^2 (+C)$	A1A1		$-\frac{1}{y}$; $3x^2$
	$x=2 \quad y=1 \quad C=-13$	M1 A1		Use (2,1) to find a constant. CAO
	$y = \frac{1}{13-3x^2}$	A1	6	CAO OE
Total			6	

1(a)(i)	$\frac{dx}{dt} = 2, \quad \frac{dy}{dt} = -8t$	B1, B1	2	CAO
(ii)	$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{-8t}{2} = -4t$	M1 A1F	2	Chain rule in correct form ft on sign coefficient errors (not power of t)
(b)	$m_T = -4, \quad m_N = \frac{1}{4}$ $x=3 \quad y=-3$ $\frac{y-3}{x-3} = \frac{1}{4} \Rightarrow \frac{y+3}{x-3} = \frac{1}{4}$	B1F, B1F M1 A1	 4	ft on $\frac{dy}{dx}$ if $f(t)$ Use candidate's (x, y) and m_N Any correct form; ISW; CAO
(c)	$t = \frac{x-1}{2}$ $y = 1 - 4\left(\frac{x-1}{2}\right)^2$	M1 M1A1	3	Substitute for t Simplification not required but CAO Or equivalent methods / forms: $y = 2x - x^2, \quad t^2 = \frac{1-y}{4},$ $\left(\frac{x-1}{2}\right)^2 = \frac{1-y}{4}$
Total			11	

8(a)(i)	$\int \frac{dy}{y} = \int \sin t \, dt$	M1		Attempt to separate and integrate
	$\ln y = -\cos t + C$	A1,A1		A1 for $\ln y$; A1 for $-\cos t$; condone missing C
	$y = Ae^{-\cos t}$	A1	4	A present; or $y = e^{-\cos t + C}$

5(a)	$x = 1, 5a^2 - a - 4 = 0$ $(5a+4)(a-1) = 0, a = 1$	M1 A1	2	condone y for a AG – be convinced, both factors seen or $a = -\frac{4}{5}$ or $1 \Rightarrow a = 1$ A0 for 2 positive roots (substitute $(1, 1) \Rightarrow 5 = 5$ no marks)
(b)	$\frac{dy}{dx} + 4$ $= 10xy^2 + 10x^2y \frac{dy}{dx}$ $x = 1, y = 1 \quad \frac{dy}{dx} + 4 = 10 + 10 \frac{dy}{dx}$ $\frac{dy}{dx} = -\frac{6}{9} = \left(-\frac{2}{3}\right)$ Alt (for last two marks) $\frac{dy}{dx} = \frac{10xy^2 - 4}{1 - 10x^2y}$	B1B1 M1 M1 A1 M1 A1 (M1)	7	(Ignore ' $\frac{dy}{dx}$ ' if not used, otherwise loses final A1) attempt product rule, see two terms added chain rule, $\frac{dy}{dx}$ attached to one term only condone 5×2 for 10 two terms, or more, in $\frac{dy}{dx}$ CSO find $\frac{dy}{dx}$ in terms of x, y and substitute $x = 1, y = 1$ must be from expression with two terms or more in $\frac{dy}{dx}$
(c)	$(1, 1) \Rightarrow \frac{10-4}{1-10} = -\frac{6}{9}$ $\frac{y-1}{x-1} = -\frac{2}{3}$ (OE)	(A1) B1F	1	fit on gradient ISW after any correct form
Total			10	

6(a)(i)	$\frac{dx}{d\theta} = -\sin\theta$ $\frac{dy}{d\theta} = 2\cos 2\theta$	B1 B1	2	
(ii)	$\frac{dy}{dx} = -\frac{2\cos 2\theta}{\sin\theta}$, $\frac{dy}{dx} = -\frac{2\cos\frac{\pi}{3}}{\sin\frac{\pi}{6}} = -2$	M1		use chain rule $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$
(b)	$y = 2\sin\theta\cos\theta = 2\sqrt{1-\cos^2\theta}\cos\theta$	A1	2	substitute $\theta = \frac{\pi}{6}$
	$y = 2\sqrt{1-x^2}x$	B1		use $\sin 2\theta = 2\sin\theta\cos\theta$
	$y^2 = 4x^2(1-x^2)$	B1		use $\sin^2\theta = 1-\cos^2\theta$
	Alt	M1		$\sin\theta, \cos\theta$ in terms of x
	$y^2 = \sin^2 2\theta = (2\sin\theta\cos\theta)^2$	A1	4	all correct CSO
	$= (4)\sin^2\theta\cos^2\theta = (4)(1-\cos^2\theta)\cos^2\theta$	(B1)		use of double angle formula
	$= (4)(1-x^2)x^2$	(B1)		use of $s^2+c^2=1$ to eliminate $\sin\theta$
	$= 4(1-x^2)x^2$	(M1)		Substitute $\cos\theta$ for x
		(A1)	(4)	CSO
Total			8	

8(a)	$\int \frac{1}{\sqrt{1+2y}} dy = \int \frac{1}{x^2} dx$	M1		attempt to separate and integrate
	$\int \frac{1}{\sqrt{1+2y}} dy = k\sqrt{1+2y}$	m1		
	$\sqrt{1+2y} = -\frac{1}{x} (+c)$	A1		OE A1 for $\sqrt{1+2y}$ depends on both Ms
	$x=1, y=4 \Rightarrow c=4$	A1		A1 for $-\frac{1}{x}$ depends on first M1 only
		m1		$+c$ must be seen on previous line
		A1F	6	ft on k and $\pm\frac{1}{x}$ only
(b)	$1+2y = \left(4-\frac{1}{x}\right)^2$	m1		need $k\sqrt{1+2y} = 'x$ expression with $+c'$
	$2y = 15 + \frac{1}{x^2} - \frac{8}{x}$	A1	2	and attempt to square both sides
				terms on RHS in any order
				AG – be convinced CSO
Total			8	