DIFFERENTIATION — Answers

1 **a** $f'(x) = 24 + 6x - 3x^2$

b $24 + 6x - 3x^2 \geq 0$

$x^2 - 2x - 8 \leq 0$

$(x + 2)(x - 4) \leq 0$

$-2 \leq x \leq 4$

2 **a** $(-2, 30) \Rightarrow 30 = -8 + 4a + 48 + b$

$\therefore\ 4a + b + 10 = 0$

b $\frac{dy}{dx} = 3x^2 + 2ax - 24$

SP at P $\therefore\ \frac{dy}{dx} = 0$

$\Rightarrow\ 12 - 4a - 24 = 0$

$a = -3,\ b = 2$

c $3x^2 - 6x - 24 = 0$

$3(x + 2)(x - 4) = 0$

$x = -2$ (at P) or 4

other SP $(4, -78)$

3 **a** $f'(x) = 2x - 16x^{-2}$

b SP: $2x - 16x^{-2} = 0$

$x^3 = 8$

$x = 2$

$\therefore\ (2, 12)$

$f''(x) = 2 + 32x^{-3}$

$f''(2) = 6$

$f''(x) > 0\ \therefore$ minimum

4 **a** area $= (2 \times \frac{1}{2}r^2\theta) + \frac{1}{2}r^2(3\theta) = 25$

$\therefore\ \frac{5}{2}r^2\theta = 25,\ \ \theta = \frac{10}{r^2}$

b $P = 2r + (2 \times r\theta) + r(3\theta) = 2r + 5r\theta$

$= 2r + 5r(\frac{10}{r^2}) = 2r + \frac{50}{r}$

c $\frac{dP}{dr} = 2 - 50r^{-2}$

SP: $2 - 50r^{-2} = 0$

$r^2 = 25$

$r = 5$

d min $P = 20$

e $\frac{d^2P}{dr^2} = 100r^{-3}$, when $r = 5$, $\frac{d^2P}{dr^2} = 0.8$

$\frac{d^2P}{dr^2} > 0\ \therefore$ minimum

5 **a** $2x - x^{\frac{3}{2}} = 0$

$x(2 - x^{\frac{1}{2}}) = 0$

$x = 0$ or $x^{\frac{1}{2}} = 2 \Rightarrow x = 4$

$\therefore$ (0, 0) and (4, 0)

b $\frac{dy}{dx} = 2 - \frac{3}{2}x^{\frac{1}{2}}$

SP: $2 - \frac{3}{2}x^{\frac{1}{2}} = 0$

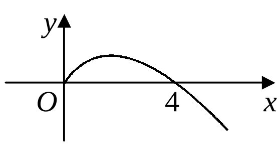
$x^{\frac{1}{2}} = \frac{4}{3}$

$x = \frac{16}{9}$

$\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$, when $x = \frac{16}{9}$, $\frac{d^2y}{dx^2} = -\frac{9}{16}$

$\frac{d^2y}{dx^2} < 0\ \therefore$ maximum

c



6 **a** $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$\therefore\ (-1, 3)$ and $(1, -1)$

b $PQ^2 = 2^2 + 4^2 = 20$

$\therefore\ PQ = \sqrt{20} = 2\sqrt{5}$

7 **a** $2x - 5 + \frac{2}{x} = 0$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$

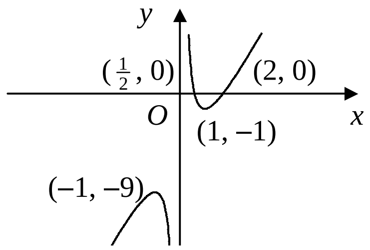
b $f'(x) = 2 - 2x^{-2}$

$\therefore\ 2 - 2x^{-2} = 0$

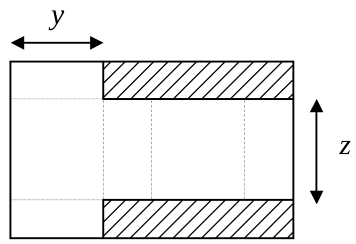
$x^2 = 1$

$x = \pm 1$

c



8 **a**



$2x + z = 25$

$2x + 2y = 40$

$\therefore$ length and width $(25 - 2x)$ and $(20 - x)$

b volume $= x(25 - 2x)(20 - x)$

$= x(500 - 65x + 2x^2)$

$= 2x^3 - 65x^2 + 500x$

c $\frac{dV}{dx} = 6x^2 - 130x + 500$

SP: $6x^2 - 130x + 500 = 0$

$2(3x - 50)(x - 5) = 0$

$x = 5, \frac{50}{3}$

$2x < 25\ \therefore\ x < 12.5$

$\therefore\ x = 5$

d max volume = 1125 cm^3

$\frac{d^2V}{dx^2} = 12x - 130$

when $x = 5$, $\frac{d^2V}{dx^2} = -70$

$\frac{d^2V}{dx^2} < 0\ \therefore$ maximum

9 **a** $\frac{dy}{dx} = 9 + 6x - 3x^2$

SP: $9 + 6x - 3x^2 = 0$

$-3(x + 1)(x - 3) = 0$

$x = -1, 3$

$\therefore$ $(-1, -3)$ and $(3, 29)$

b $\frac{d^2y}{dx^2} = 6 - 6x$

$(-1, -3)$: $\frac{d^2y}{dx^2} = 12\ \therefore$ minimum

$(3, 29)$: $\frac{d^2y}{dx^2} = -12\ \therefore$ maximum

c $-3 < k < 29$

10 **a** $f(-1) = 15$

$\therefore\ -4 + a + 12 + b = 15$

$a + b = 7$ (1)

b $f(2) = 42$

$\therefore\ 32 + 4a - 24 + b = 42$

$4a + b = 34$ (2)

(2) − (1) $3a = 27$

$\therefore\ a = 9,\ b = -2$

c $f(x) = 4x^3 + 9x^2 - 12x - 2$

$f'(x) = 12x^2 + 18x - 12$

SP: $12x^2 + 18x - 12 = 0$

$2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$x = -2, \frac{1}{2}$

$\therefore$ $(-2, 26)$ and $(\frac{1}{2}, -\frac{21}{4})$