DIFFERENTIATION

1 $$f(x) \equiv 7 + 24x + 3x^2 - x^3.$$

a Find $f'(x)$. **(2)**

b Find the set of values of x for which $f(x)$ is increasing. **(4)**

2 The curve with equation $y = x^3 + ax^2 - 24x + b$, where a and b are constants, passes through the point P $(-2, 30)$.

a Show that $4a + b + 10 = 0$. **(2)**

Given also that P is a stationary point of the curve,

b find the values of a and b, **(4)**

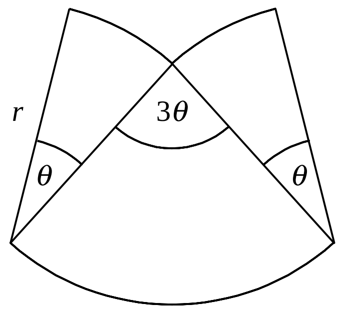
c find the coordinates of the other stationary point on the curve. **(3)**

3 $$f(x) \equiv x^2 + \frac{16}{x}, \quad x \neq 0.$$

a Find $f'(x)$. **(2)**

b Find the coordinates of the stationary point of the curve $y = f(x)$ and determine its nature. **(6)**

4



The diagram shows a design to be used on a new brand of cat-food. The design consists of three circular sectors, each of radius r cm. The angle of two of the sectors is θ radians and the angle of the third sector is 3θ radians as shown.

Given that the area of the design is 25 cm^2,

a show that $\theta = \dfrac{10}{r^2}$, **(3)**

b find the perimeter of the design, P cm, in terms of r. **(3)**

Given that r can vary,

c find the value of r for which P takes it minimum value, **(4)**

d find the minimum value of P, **(1)**

e justify that the value you have found is a minimum. **(2)**

5 The curve C has the equation

$$y = 2x - x^{\frac{3}{2}}, \quad x \geq 0.$$

a Find the coordinates of any points where C meets the x-axis. **(3)**

b Find the x-coordinate of the stationary point on C and determine whether it is a maximum or a minimum point. **(6)**

c Sketch the curve C. **(2)**

6 The curve $y = x^3 - 3x + 1$ is stationary at the points P and Q.

a Find the coordinates of the points P and Q. **(5)**

b Find the length of PQ in the form $k\sqrt{5}$. **(3)**

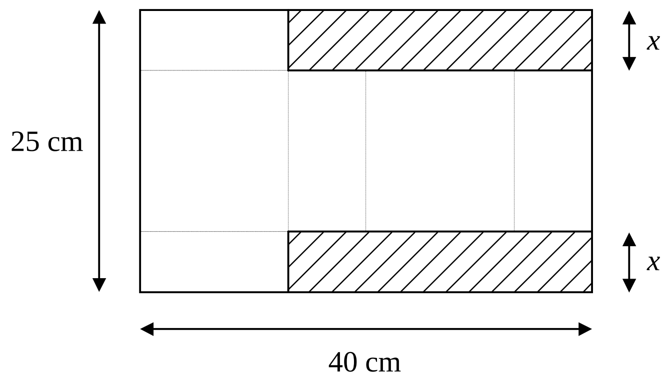
7
$$f(x) \equiv 2x - 5 + \frac{2}{x}, \quad x \neq 0.$$

a Solve the equation $f(x) = 0$. **(4)**

b Solve the equation $f'(x) = 0$. **(4)**

c Sketch the curve $y = f(x)$, showing the coordinates of any turning points and of any points where the curve crosses the coordinate axes. **(3)**

8



Two identical rectangles of width x cm are removed from a rectangular piece of card measuring 25 cm by 40 cm as shown in the diagram above. The remaining card is the net of a cuboid of height x cm.

a Find expressions in terms of x for the length and width of the base of the cuboid formed from the net. **(3)**

b Show that the volume of the cuboid is $(2x^3 - 65x^2 + 500x)$ cm^3. **(2)**

c Find the value of x for which the volume of the cuboid is a maximum. **(5)**

d Find the maximum volume of the cuboid and show that it is a maximum. **(3)**

9 **a** Find the coordinates of the stationary points on the curve
$$y = 2 + 9x + 3x^2 - x^3.$$ **(6)**

b Determine whether each stationary point is a maximum or minimum point. **(2)**

c State the set of values of k for which the equation
$$2 + 9x + 3x^2 - x^3 = k$$
has three solutions. **(2)**

10
$$f(x) = 4x^3 + ax^2 - 12x + b.$$

Given that a and b are constants and that when $f(x)$ is divided by $(x + 1)$ there is a remainder of 15,

a find the value of $(a + b)$. **(2)**

Given also that when $f(x)$ is divided by $(x - 2)$ there is a remainder of 42,

b find the values of a and b, **(3)**

c find the coordinates of the stationary points of the curve $y = f(x)$. **(6)**