DIFFERENTIATION

1 $\mathrm{f}(x) = (x + 1)(x - 2)^2.$

a Sketch the curve $y = \mathrm{f}(x)$, showing the coordinates of any points where the curve meets the coordinate axes. **(3)**

b Find $\mathrm{f}'(x)$. **(4)**

c Show that the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = 1$ has the equation

$$y = 5 - 3x.$$ **(3)**

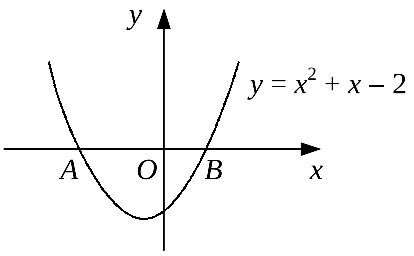
2 The curve C has the equation $y = x - 3x^{\frac{1}{2}} + 3$ and passes through the point P (4, 1).

a Show that the tangent to C at P passes through the origin. **(5)**

The normal to C at P crosses the y-axis at the point Q.

b Find the area of triangle OPQ, where O is the origin. **(4)**

3



The diagram shows the curve $y = x^2 + x - 2$. The curve crosses the x-axis at the points A $(a, 0)$ and B $(b, 0)$ where $a < b$.

a Find the values of a and b. **(3)**

b Show that the normal to the curve at A has the equation

$$x - 3y + 2 = 0.$$ **(5)**

The tangent to the curve at B meets the normal to the curve at A at the point C.

c Find the exact coordinates of C. **(4)**

4 Given that $y = \frac{x^2 - 6x - 3}{3x^{\frac{1}{2}}}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $\frac{(x+a)^2}{bx^{\frac{3}{2}}}$, where a and b are integers to be found. **(6)**

5 The point A lies on the curve $y = \frac{12}{x^2}$ and the x-coordinate of A is 2.

a Find an equation of the tangent to the curve at A. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

b Verify that the point where the tangent at A intersects the curve again has the coordinates $(-1, 12)$. **(3)**

6 A curve has the equation $y = 2 + 3x + kx^2 - x^3$ where k is a constant.

Given that the gradient of the curve is -6 at the point P where $x = -1$,

a find the value of k. **(4)**

Given also that the tangent to the curve at the point Q is parallel to the tangent at P,

b find the length PQ, giving your answer in the form $k\sqrt{5}$. **(5)**

7 Differentiate $x^2 + \frac{1}{2x}$ with respect to x. **(3)**

8 A curve has the equation $y = 2x^2 - 7x + 1$ and the point A on the curve has x-coordinate 2.

a Find an equation of the tangent to the curve at A. **(4)**

The normal to the curve at the point B is parallel to the tangent at A.

b Find the coordinates of B. **(3)**

9 $$y = x^2 + 3x^{\frac{1}{2}}.$$

a Find $\frac{dy}{dx}$. **(2)**

b Show that $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x = 0$. **(4)**

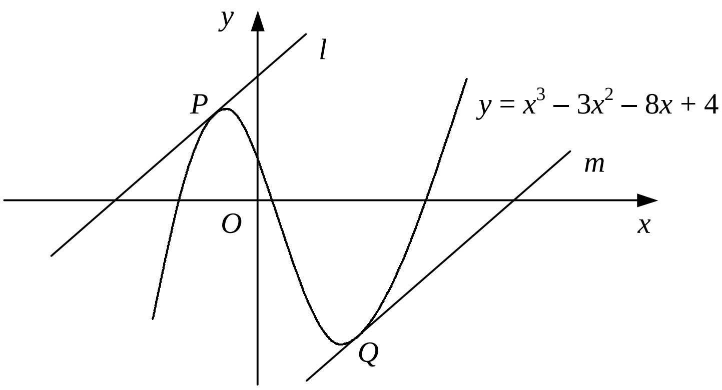
10 A curve has the equation $y = 2 + \frac{4}{x}$.

a Find an equation of the normal to the curve at the point M (4, 3). **(5)**

The normal to the curve at M intersects the curve again at the point N.

b Find the coordinates of the point N. **(5)**

11



The diagram shows the curve with equation $y = x^3 - 3x^2 - 8x + 4$.

The straight line l is the tangent to the curve at the point P (–1, 8).

a Find an equation of line l. **(4)**

The straight line m is parallel to l and is the tangent to the curve at the point Q.

b Find an equation of line m. **(4)**

c Find an equation of the normal to the curve at P. **(2)**

d Hence, or otherwise, show that the distance between lines l and m is $16\sqrt{2}$. **(4)**

12 A curve has the equation $y = \sqrt{x}\,(k - x)$, where k is a constant.

Given that the gradient of the curve is $\sqrt{2}$ at the point P where $x = 2$,

a find the value of k, **(5)**

b show that the normal to the curve at P has the equation

$$x + \sqrt{2}\,y = c,$$

where c is an integer to be found. **(5)**