DIFFERENTIATION — Answers

1 **a** $f'(x) = 6x^2 + 10x$

b $6x^2 + 10x \geq 0$

$2x(3x + 5) \geq 0$

$x \leq -\frac{5}{3}$ and $x \geq 0$

2 **a** $\frac{dy}{dx} = 3x^2 - 2x + 2$

at $(1, -2)$, grad $= 3$

$\therefore\ y + 2 = 3(x - 1)$

$3x - y - 5 = 0$

b SP when $3x^2 - 2x + 2 = 0$

$b^2 - 4ac = 4 - 24 = -20$

$b^2 - 4ac < 0\ \therefore$ no real roots

$\therefore$ no stationary points

3 **a** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-2}$

$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} + 8x^{-3}$

b SP: $\frac{1}{2}x^{-\frac{1}{2}} - 4x^{-2} = 0$

$\frac{1}{2}x^{-2}(x^{\frac{3}{2}} - 8) = 0$

$x^{\frac{3}{2}} = 8$

$x = 4$

$\therefore\ (4, 3)$

when $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{32}$

$\frac{d^2y}{dx^2} > 0\ \therefore$ minimum

4 **a** $y = 0 \Rightarrow x(x + 3)^2 = 0$

$x = -3, 0$

$\therefore\ (-3, 0), (0, 0)$

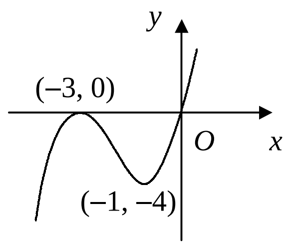
b $f'(x) = 3x^2 + 12x + 9$

decreasing when $3x^2 + 12x + 9 \leq 0$

$3(x + 3)(x + 1) \leq 0$

$\therefore\ -3 \leq x \leq -1$

c



5 **a** $\frac{dh}{dt} = 8t^3 - 24t^2 + 16t$

b when $t = 0.25$,

$\frac{dh}{dt} = 2.625$ cm per second

c SP: $8t^3 - 24t^2 + 16t = 0$

$8t(t - 1)(t - 2) = 0$

$t = 0, 1, 2$

from graph, max when $t = 1$

$\therefore$ max height $= 3$ cm

6 **a** $\frac{dy}{dx} = 3x^2 + 6kx - 9k^2$

stationary when $3x^2 + 6kx - 9k^2 = 0$

$\Rightarrow\ x^2 + 2kx - 3k^2 = 0$

b $(x + 3k)(x - k) = 0$

$x = -3k, k$

when $x = k$, $y = k^3 + 3k^3 - 9k^3 = -5k^3$

$\therefore$ stationary at $(k, -5k^3)$

c when $x = -3k$,

$y = -27k^3 + 27k^3 + 27k^3 = 27k^3$

$\therefore\ (-3k, 27k^3)$

7 **a** $V = \frac{1}{2}x^2 \sin 60° \times l$

$= \frac{1}{2}x^2 l \times \frac{\sqrt{3}}{2} = 250$

$\therefore\ l = \frac{1000}{\sqrt{3}x^2}$ or $\frac{1000\sqrt{3}}{3x^2}$

b $A = (2 \times \frac{\sqrt{3}}{4}x^2) + 3xl$

$= \frac{\sqrt{3}}{2}x^2 + (3x \times \frac{1000\sqrt{3}}{3x^2})$

$= \frac{\sqrt{3}}{2}(x^2 + \frac{2000}{x})$

c $\frac{dA}{dx} = \frac{\sqrt{3}}{2}(2x - 2000x^{-2})$

SP: $\frac{\sqrt{3}}{2}(2x - 2000x^{-2}) = 0$

$x^3 = 1000$

$x = 10$

d min $A = 150\sqrt{3}$

e $\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}(2 + 4000x^{-3})$

when $x = 10$, $\frac{d^2A}{dx^2} = 3\sqrt{3}$

$\frac{d^2A}{dx^2} > 0\ \therefore$ minimum

8 **a** $f'(x) = 3x^2 + 8x + k$

for 2 SPs, $f'(x) = 0$ has 2 distinct roots

$\therefore\ b^2 - 4ac > 0$

$64 - 12k > 0$

$k < \frac{16}{3}$

b SP: $3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$

$x = -3, \frac{1}{3}$

$\therefore$ $(-3, 19)$ and $(\frac{1}{3}, \frac{13}{27})$

9 **a** $x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}} = 0$

$x - 4x^{\frac{1}{2}} + 3 = 0$

$(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 3) = 0$

$x^{\frac{1}{2}} = 1, 3$

$x = 1, 9$

$\therefore$ (1, 0) and (9, 0)

b $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

SP: $\frac{1}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} = 0$

$\frac{1}{2}x^{-\frac{3}{2}}(x - 3) = 0$

$x = 3$

$y = \sqrt{3} - 4 + \frac{3}{\sqrt{3}} = 2\sqrt{3} - 4$

$\therefore$ $(3, 2\sqrt{3} - 4)$

10 **a** $f(-1) = -1 - 3 + 4 = 0$

$\therefore$ $(x + 1)$ is a factor

b

$$
\begin{array}{r|l}
 & x^2 - 4x + 4 \\
x + 1 & x^3 - 3x^2 + 0x + 4 \\
 & x^3 + x^2 \\
 & \quad -4x^2 + 0x \\
 & \quad -4x^2 - 4x \\
 & \qquad 4x + 4 \\
 & \qquad 4x + 4
\end{array}
$$

$\therefore$ $f(x) \equiv (x + 1)(x^2 - 4x + 4)$

$f(x) \equiv (x + 1)(x - 2)^2$

c (2, 0), as $(x - 2)$ is a repeated factor of $f(x)$ so x-axis is a tangent at (2, 0)

d $f'(x) = 3x^2 - 6x$

SP: $3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0, 2$

$\therefore$ (0, 4) is other turning point