# DIFFERENTIATION

**1** $$f(x) \equiv 2x^3 + 5x^2 - 1.$$

**a** Find $f'(x)$.

**b** Find the set of values of $x$ for which $f(x)$ is increasing.

**2** The curve $C$ has the equation $y = x^3 - x^2 + 2x - 4$.

**a** Find an equation of the tangent to $C$ at the point $(1, -2)$. Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**b** Prove that the curve $C$ has no stationary points.

**3** A curve has the equation $y = \sqrt{x} + \frac{4}{x}$.

**a** Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

**b** Find the coordinates of the stationary point of the curve and determine its nature.

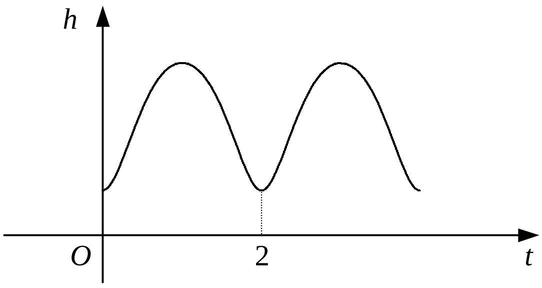
**4** $$f(x) \equiv x^3 + 6x^2 + 9x.$$

**a** Find the coordinates of the points where the curve $y = f(x)$ meets the $x$-axis.

**b** Find the set of values of $x$ for which $f(x)$ is decreasing.

**c** Sketch the curve $y = f(x)$, showing the coordinates of any stationary points.

**5**




The graph shows the height, $h$ cm, of the letters on a website advert $t$ seconds after the advert appears on the screen.

For $t$ in the interval $0 \le t \le 2$, $h$ is given by the equation

$$h = 2t^4 - 8t^3 + 8t^2 + 1.$$

For larger values of $t$, the variation of $h$ over this interval is repeated every 2 seconds.

**a** Find $\frac{dh}{dt}$ for $t$ in the interval $0 \le t \le 2$.

**b** Find the rate at which the height of the letters is increasing when $t = 0.25$

**c** Find the maximum height of the letters.

**6** The curve $C$ has the equation $y = x^3 + 3kx^2 - 9k^2x$, where $k$ is a non-zero constant.

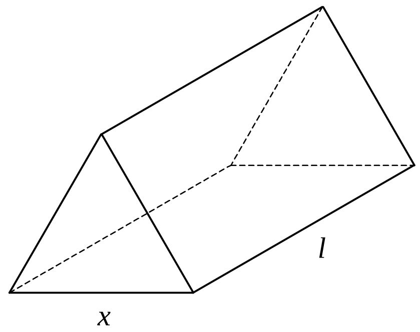
**a** Show that $C$ is stationary when

$$x^2 + 2kx - 3k^2 = 0.$$

**b** Hence, show that $C$ is stationary at the point with coordinates $(k, -5k^3)$.

**c** Find, in terms of $k$, the coordinates of the other stationary point on $C$.

**7**




The diagram shows a solid triangular prism. The cross-section of the prism is an equilateral triangle of side $x$ cm and the length of the prism is $l$ cm.

Given that the volume of the prism is 250 $cm^3$,

**a** find an expression for $l$ in terms of $x$,

**b** show that the surface area of the prism, $A$ $cm^2$, is given by

$$A = \frac{\sqrt{3}}{2}(x^2 + \frac{2000}{x}).$$

Given that $x$ can vary,

**c** find the value of $x$ for which $A$ is a minimum,

**d** find the minimum value of $A$ in the form $k\sqrt{3}$,

**e** justify that the value you have found is a minimum.

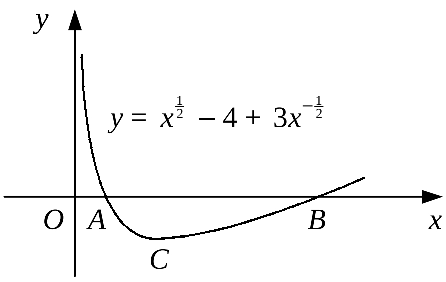
**8** $$f(x) \equiv x^3 + 4x^2 + kx + 1.$$

**a** Find the set of values of the constant $k$ for which the curve $y = f(x)$ has two stationary points.

Given that $k = -3$,

**b** find the coordinates of the stationary points of the curve $y = f(x)$.

**9**




The diagram shows the curve with equation $y = x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}}$. The curve crosses the $x$-axis at the points $A$ and $B$ and has a minimum point at $C$.

**a** Find the coordinates of $A$ and $B$.

**b** Find the coordinates of $C$, giving its $y$-coordinate in the form $a\sqrt{3} + b$, where $a$ and $b$ are integers.

**10** $$f(x) = x^3 - 3x^2 + 4.$$

**a** Show that $(x + 1)$ is a factor of f($x$).

**b** Fully factorise f($x$).

**c** Hence state, with a reason, the coordinates of one of the turning points of the curve $y = f(x)$.

**d** Using differentiation, find the coordinates of the other turning point of the curve $y = f(x)$.