


DIFFERENTIATION
Answers

- 1**
- a** $\frac{dy}{dx} = 2x + 6$
 $2x + 6 = 0$
 $x = -3$
- b** $\frac{dy}{dx} = 8x + 2$
 $8x + 2 = 0$
 $x = -\frac{1}{4}$
- c** $\frac{dy}{dx} = 3x^2 - 12$
 $3x^2 - 12 = 0$
 $x^2 = 4$
 $x = \pm 2$
- d** $\frac{dy}{dx} = 18x - 3x^2$
 $18x - 3x^2 = 0$
 $3x(6 - x) = 0$
 $x = 0, 6$
- e** $\frac{dy}{dx} = 3x^2 - 10x + 3$
 $3x^2 - 10x + 3 = 0$
 $(3x - 1)(x - 3) = 0$
 $x = \frac{1}{3}, 3$
- f** $\frac{dy}{dx} = 1 - 9x^{-2}$
 $1 - 9x^{-2} = 0$
 $x^2 = 9$
 $x = \pm 3$
- g** $y = x^3 - 3x^2 + 3x - 9$
 $\frac{dy}{dx} = 3x^2 - 6x + 3$
 $3x^2 - 6x + 3 = 0$
 $3(x - 1)^2 = 0$
 $x = 1$
- h** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2$
 $\frac{1}{2}x^{-\frac{1}{2}} - 2 = 0$
 $x^{-\frac{1}{2}} = 4$
 $x = \frac{1}{16}$
- 2**
- a** $f'(x) = 4x + 2$
 $\therefore 4x + 2 \geq 0$
 $x \geq -\frac{1}{2}$
- b** $f'(x) = 6x - 6x^2$
 $\therefore 6x - 6x^2 \geq 0$
 $6x(1 - x) \geq 0$
 $0 \leq x \leq 1$
- c** $f'(x) = 9x^2 - 1$
 $\therefore 9x^2 - 1 \geq 0$
 $x^2 \geq \frac{1}{9}$
 $x \leq -\frac{1}{3}$ and $x \geq \frac{1}{3}$
- d** $f'(x) = 3x^2 + 12x - 15$
 $\therefore 3x^2 + 12x - 15 \geq 0$
 $3(x + 5)(x - 1) \geq 0$
 $x \leq -5$ and $x \geq 1$
- e** $f(x) = x^3 - 12x^2 + 36x$
 $f'(x) = 3x^2 - 24x + 36$
 $\therefore 3x^2 - 24x + 36 \geq 0$
 $3(x - 2)(x - 6) \geq 0$
 $x \leq 2$ and $x \geq 6$
- f** $f'(x) = 2 - 8x^{-2}$
 $\therefore 2 - 8x^{-2} \geq 0$
 $x^2 \geq 4$
 $x \leq -2$ and $x \geq 2$
- 3**
- a** $f'(x) = 3x^2 + 4x$
 $\therefore 3x^2 + 4x \leq 0$
 $x(3x + 4) \leq 0$
 $-\frac{4}{3} \leq x \leq 0$
- b** $f'(x) = 27 - 3x^2$
 $\therefore 27 - 3x^2 \leq 0$
 $x^2 \geq 9$
 $x \leq -3$ and $x \geq 3$
- c** $f(x) = 2x^3 - x^2 - 4x + 2$
 $f'(x) = 6x^2 - 2x - 4$
 $\therefore 6x^2 - 2x - 4 \leq 0$
 $2(3x + 2)(x - 1) \leq 0$
 $-\frac{2}{3} \leq x \leq 1$
- 4**
- a** $(x + 1)$ factor $\therefore f(-1) = 0$
 $\therefore -1 + k + 3 = 0$
 $k = -2$
- b** $f'(x) = 3x^2 - 4x$
 $\therefore 3x^2 - 4x \geq 0$
 $x(3x - 4) \geq 0$
 $x \leq 0$ and $x \geq \frac{4}{3}$

- 5**
- a** $\frac{dy}{dx} = 2x + 2$
 SP: $2x + 2 = 0$
 $x = -1$
 $\therefore (-1, -1)$
- b** $\frac{dy}{dx} = 10x - 4$
 SP: $10x - 4 = 0$
 $x = \frac{2}{5}$
 $\therefore (\frac{2}{5}, \frac{1}{5})$
- c** $\frac{dy}{dx} = 3x^2 - 3$
 SP: $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = \pm 1$
 $\therefore (-1, 6), (1, 2)$
- d** $\frac{dy}{dx} = 12x^2 + 6x$
 SP: $12x^2 + 6x = 0$
 $6x(2x + 1) = 0$
 $x = -\frac{1}{2}, 0$
 $\therefore (-\frac{1}{2}, \frac{9}{4}), (0, 2)$
- e** $\frac{dy}{dx} = 2 - 8x^{-2}$
 SP: $2 - 8x^{-2} = 0$
 $x^2 = 4$
 $x = \pm 2$
 $\therefore (-2, -5), (2, 11)$
- f** $\frac{dy}{dx} = 3x^2 - 18x - 21$
 SP: $3x^2 - 18x - 21 = 0$
 $3(x + 1)(x - 7) = 0$
 $x = -1, 7$
 $\therefore (-1, 22), (7, -234)$
- g** $\frac{dy}{dx} = -x^{-2} - 8x$
 SP: $-x^{-2} - 8x = 0$
 $x^3 = -\frac{1}{8}$
 $x = -\frac{1}{2}$
 $\therefore (-\frac{1}{2}, -3)$
- h** $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$
 SP: $3x^{\frac{1}{2}} - 6 = 0$
 $x^{\frac{1}{2}} = 2$
 $x = 4$
 $\therefore (4, -8)$
- i** $\frac{dy}{dx} = 6x^{-\frac{1}{3}} - 2$
 SP: $6x^{-\frac{1}{3}} - 2 = 0$
 $x^{-\frac{1}{3}} = \frac{1}{3}$
 $x = \frac{1}{27}$
 $\therefore (\frac{1}{27}, 5\frac{25}{27})$
- 6**
- a** $\frac{dy}{dx} = 4 - 2x$
 SP: $4 - 2x = 0$
 $x = 2$
 $\frac{d^2y}{dx^2} = -2$
 $(2, 9): \text{max}$
- b** $\frac{dy}{dx} = 3x^2 - 3$
 SP: $3x^2 - 3 = 0$
 $x^2 = 1$
 $x = \pm 1$
 $\frac{d^2y}{dx^2} = 6x$
 $(-1, 2): \frac{d^2y}{dx^2} = -6, \text{max}$
 $(1, -2): \frac{d^2y}{dx^2} = 6, \text{min}$
- c** $\frac{dy}{dx} = 3x^2 + 18x$
 SP: $3x^2 + 18x = 0$
 $3x(x + 6) = 0$
 $x = -6, 0$
 $\frac{d^2y}{dx^2} = 6x + 18$
 $(-6, 100): \frac{d^2y}{dx^2} = -18, \text{max}$
 $(0, -8): \frac{d^2y}{dx^2} = 18, \text{min}$
- d** $\frac{dy}{dx} = 3x^2 - 12x - 36$
 SP: $3x^2 - 12x - 36 = 0$
 $3(x + 2)(x - 6) = 0$
 $x = -2, 6$
 $\frac{d^2y}{dx^2} = 6x - 12$
 $(-2, 55): \frac{d^2y}{dx^2} = -24, \text{max}$
 $(6, -201): \frac{d^2y}{dx^2} = 24, \text{min}$
- e** $\frac{dy}{dx} = 4x^3 - 16x$
 SP: $4x^3 - 16x = 0$
 $4x(x^2 - 4) = 0$
 $x = 0, \pm 2$
 $\frac{d^2y}{dx^2} = 12x^2 - 16$
 $(-2, -18): \frac{d^2y}{dx^2} = 32, \text{min}$
 $(0, -2): \frac{d^2y}{dx^2} = -16, \text{max}$
 $(2, -18): \frac{d^2y}{dx^2} = 32, \text{min}$
- f** $\frac{dy}{dx} = 9 - 4x^{-2}$
 SP: $9 - 4x^{-2} = 0$
 $x^2 = \frac{4}{9}$
 $x = \pm \frac{2}{3}$
 $\frac{d^2y}{dx^2} = 8x^{-3}$
 $(-\frac{2}{3}, -12): \frac{d^2y}{dx^2} = -27, \text{max}$
 $(\frac{2}{3}, 12): \frac{d^2y}{dx^2} = 27, \text{min}$

$$\mathbf{g} \quad \frac{dy}{dx} = 1 - 3x^{-\frac{1}{2}}$$

$$\text{SP: } 1 - 3x^{-\frac{1}{2}} = 0$$

$$x^{-\frac{1}{2}} = \frac{1}{3}$$

$$x = 9$$

$$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{3}{2}}$$

$$(9, -9): \frac{d^2y}{dx^2} = \frac{1}{18}, \text{ min}$$

$$\mathbf{h} \quad \frac{dy}{dx} = -8 + 14x - 6x^2$$

$$\text{SP: } -8 + 14x - 6x^2 = 0$$

$$-2(3x - 4)(x - 1) = 0$$

$$x = 1, \frac{4}{3}$$

$$\frac{d^2y}{dx^2} = 14 - 12x$$

$$(1, 0): \frac{d^2y}{dx^2} = 2, \text{ min}$$

$$\left(\frac{4}{3}, \frac{1}{27}\right): \frac{d^2y}{dx^2} = -2, \text{ max}$$

$$\mathbf{i} \quad y = \frac{1}{2}x^2 + 8x^{-2}$$

$$\frac{dy}{dx} = x - 16x^{-3}$$

$$\text{SP: } x - 16x^{-3} = 0$$

$$x^4 = 16$$

$$x = \pm 2$$

$$\frac{d^2y}{dx^2} = 1 + 48x^{-4}$$

$$(-2, 4): \frac{d^2y}{dx^2} = 4, \text{ min}$$

$$(2, 4): \frac{d^2y}{dx^2} = 4, \text{ min}$$

$$\mathbf{7} \quad \mathbf{a} \quad \frac{dy}{dx} = 2x - 3x^2$$

$$\text{SP: } 2x - 3x^2 = 0$$

$$x(2 - 3x) = 0$$

$$x = 0, \frac{2}{3}$$

$$\frac{d^2y}{dx^2} = 2 - 6x$$

$$(0, 0): \frac{d^2y}{dx^2} = 2, \text{ min}$$

$$\left(\frac{2}{3}, \frac{4}{27}\right): \frac{d^2y}{dx^2} = -2, \text{ max}$$

$$\mathbf{b} \quad \frac{dy}{dx} = 3x^2 + 6x + 3$$

$$\text{SP: } 3x^2 + 6x + 3 = 0$$

$$3(x + 1)^2 = 0$$

$$x = -1$$

$$\frac{d^2y}{dx^2} = 6x + 6$$

$$(-1, -1): \frac{d^2y}{dx^2} = 0$$

x	< -1	-1	> -1
$\frac{dy}{dx}$	$+$	0	$+$

$\frac{dy}{dx}$	$+$	0	$+$
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$\therefore (-1, -1)$: point of inflexion

$$\mathbf{c} \quad \frac{dy}{dx} = 4x^3$$

$$\text{SP: } 4x^3 = 0$$

$$x = 0$$

$$\frac{d^2y}{dx^2} = 12x^2$$

$$(0, -2): \frac{d^2y}{dx^2} = 0$$

x	< 0	0	> 0
$\frac{dy}{dx}$	$-$	0	$+$

$\frac{dy}{dx}$	$-$	0	$+$
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$\therefore (0, -2)$: min

$$\mathbf{d} \quad \frac{dy}{dx} = -12 + 12x - 3x^2$$

$$\text{SP: } -12 + 12x - 3x^2 = 0$$

$$-3(x - 2)^2 = 0$$

$$x = 2$$

$$\frac{d^2y}{dx^2} = 12 - 6x$$

$$(2, -4): \frac{d^2y}{dx^2} = 0$$

x	< 2	2	> 2
$\frac{dy}{dx}$	$-$	0	$-$

$\frac{dy}{dx}$	$-$	0	$-$
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$\therefore (2, -4)$: point of inflexion

$$\mathbf{e} \quad \frac{dy}{dx} = 2x - 16x^{-2}$$

$$\text{SP: } 2x - 16x^{-2} = 0$$

$$x^3 = 8$$

$$x = 2$$

$$\frac{d^2y}{dx^2} = 2 + 32x^{-3}$$

$$(2, 12): \frac{d^2y}{dx^2} = 6, \text{ min}$$

$$\mathbf{f} \quad \frac{dy}{dx} = 4x^3 + 12x^2$$

$$\text{SP: } 4x^3 + 12x^2 = 0$$

$$4x^2(x + 3) = 0$$

$$x = -3, 0$$

$$\frac{d^2y}{dx^2} = 12x^2 + 24x$$

$$(-3, -28): \frac{d^2y}{dx^2} = 36, \text{ min}$$

$$(0, -1): \frac{d^2y}{dx^2} = 0$$

x	$-3 < x < 0$	0	> 0
$\frac{dy}{dx}$	$+$	0	$+$

$\frac{dy}{dx}$	$+$	0	$+$
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$\therefore (0, -1)$: point of inflexion

8 a $\frac{dy}{dx} = 3x^2 + 6x$

SP: $3x^2 + 6x = 0$

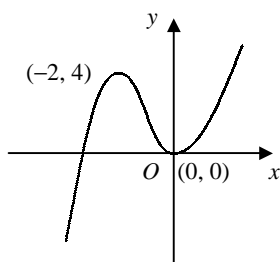
$$3x(x + 2) = 0$$

$$x = -2, 0$$

$$\frac{d^2y}{dx^2} = 6x + 6$$

$(-2, 4): \frac{d^2y}{dx^2} = -6, \text{ max}$

$(0, 0): \frac{d^2y}{dx^2} = 6, \text{ min}$



b $\frac{dy}{dx} = 1 - x^{-2}$

SP: $1 - x^{-2} = 0$

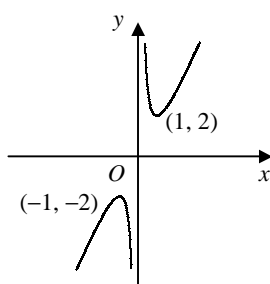
$$x^2 = 1$$

$$x = \pm 1$$

$$\frac{d^2y}{dx^2} = 2x^{-3}$$

$(-1, -2): \frac{d^2y}{dx^2} = -2, \text{ max}$

$(1, 2): \frac{d^2y}{dx^2} = 2, \text{ min}$



c $\frac{dy}{dx} = 3x^2 - 6x + 3$

SP: $3x^2 - 6x + 3 = 0$

$$3(x - 1)^2 = 0$$

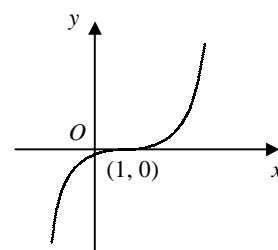
$$x = 1$$

$$\frac{d^2y}{dx^2} = 6x - 6$$

$(1, 0): \frac{d^2y}{dx^2} = 0$

x	< 1	1	> 1
$\frac{dy}{dx}$	$+$	0	$+$

$\therefore (1, 0): \text{ point of inflexion}$



d $\frac{dy}{dx} = 3 - 2x^{-\frac{1}{2}}$

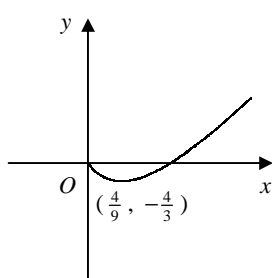
SP: $3 - 2x^{-\frac{1}{2}} = 0$

$$x^{-\frac{1}{2}} = \frac{3}{2}$$

$$x = \frac{4}{9}$$

$$\frac{d^2y}{dx^2} = x^{-\frac{3}{2}}$$

$(\frac{4}{9}, -\frac{4}{3}): \frac{d^2y}{dx^2} = \frac{27}{8}, \text{ min}$



e $\frac{dy}{dx} = 3x^2 + 8x - 3$

SP: $3x^2 + 8x - 3 = 0$

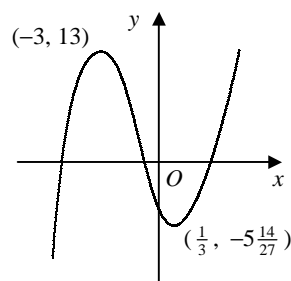
$$(3x - 1)(x + 3) = 0$$

$$x = -3, \frac{1}{3}$$

$$\frac{d^2y}{dx^2} = 6x + 8$$

$(-3, 13): \frac{d^2y}{dx^2} = -10, \text{ max}$

$(\frac{1}{3}, -5\frac{14}{27}): \frac{d^2y}{dx^2} = 10, \text{ min}$



f $y = x^4 - 8x^2 + 12$

$$\frac{dy}{dx} = 4x^3 - 16x$$

SP: $4x^3 - 16x = 0$

$$4x(x + 2)(x - 2) = 0$$

$$x = -2, 0, 2$$

$$\frac{d^2y}{dx^2} = 12x^2 - 16$$

$(-2, -4): \frac{d^2y}{dx^2} = 32, \text{ min}$

$(0, 12): \frac{d^2y}{dx^2} = -16, \text{ max}$

$(2, -4): \frac{d^2y}{dx^2} = 32, \text{ min}$

